

Benchmarks for Excellent Student  
Thinking (B.E.S.T.)

Mathematics

Access Points-Alternate Academic  
Achievement Standards (AP-AAAS)  
with Essential Understandings (EUs)

Kindergarten -12

**Kindergarten B.E.S.T. Standards Access Points  
Number Sense and Operations**

<b>MA.K.NSO.1 Develop an understanding for counting using objects in a set.</b>	
MA.K.NSO.1.1	Given a group of up to 20 objects, count the number of objects in that group and represent the number of objects with a written numeral. State the number of objects in a rearrangement of that group without recounting.
	<p><b>Access Point</b></p> <p>MA.K.NSO.1.AP.1 Given a group of up to 10 objects, count the number of objects in that group and represent the number by identifying the written numeral. Express the number of objects in a rearrangement of that group without recounting.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Express number names (rote count) up to 10.</li> <li>• Understand that counting has cardinality in the numbers (last number named when counting tells the number of objects counted).</li> <li>• Identify the written numeral when given the name of the numeral up to 10.</li> <li>• Understand that the total number of objects in a group remains the same if no objects are added to or removed from the group.</li> </ul>
MA.K.NSO.1.2	Given a number from 0 to 20, count out that many objects.
	<p><b>Access Point</b></p> <p>MA.K.NSO.1.AP.2 Given a number from 0 to 10, count out that many objects.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Express number names (rote count) up to 10 Use 1:1 correspondence to count up to 10 objects when arranged in a line or scattered formation.</li> <li>• Understand that counting has cardinality in the numbers (last number named when counting out objects tells the total number of objects counted out).</li> </ul>

MA.K.NSO.1.3	<p>Identify positions of objects within a sequence using the words “first,” “second,” “third,” “fourth” or “fifth.”</p> <p><b>Access Point</b></p> <p>MA.K.NSO.1.AP.3 Identify the “first,” “second” or “third” object within a sequence.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the positional terms “first,” “second,” and “third”.</li> </ul>
MA.K.NSO.1.4	<p>Compare the number of objects from 0 to 20 in two groups using the terms less than, equal to or greater than.</p> <p><b>Access Point</b></p> <p>MA.K.NSO.1.AP.4 Compare the number of objects from 0 to 10 in two groups to determine which group is greater or less, or if the number of objects in the two groups are equal.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Align objects 1-to-1.</li> <li>• Understand the concepts “more” or “less”.</li> <li>• Understand the concept of “greater than” as more objects, “less than” as fewer objects, and “equal to” as the same number of objects.</li> </ul>
<p><b><i>MA.K.NSO.2 Recite number names sequentially within 100 and develop an understanding for place value.</i></b></p>	
MA.K.NSO.2.1	<p>Recite the number names to 100 by ones and by tens. Starting at a given number, count forward within 100 and backward within 20.</p> <p><b>Access Point</b></p> <p>MA.K.NSO.2.AP.1 Express number names from 1 to 100 by ones and from 10 to 100 by tens. Starting at a given number, count forward to 20 and backwards within 10.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand there is a consistent order when counting.</li> <li>• Understand the concepts of “forward” and “backward”.</li> </ul>
MA.K.NSO.2.2	<p>Represent whole numbers from 10 to 20, using a unit of ten and a group of ones, with objects, drawings, and expressions or equations.</p>

	<p><b>Access Point</b></p> <p>MA.K.NSO.2.AP.2 Represent whole numbers from 10 to 19, using one group of 10 ones and some further ones, with objects, drawings or verbalization.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Express number names (rote count) from 10-19.</li> <li>• Use 1:1 correspondence to represent one group of 10 objects in a 10-frame.</li> <li>• Recognize that a full 10-frame represents 1 group of 10 ones.</li> <li>• Use 1:1 correspondence with objects to count on from a full 10-frame up to 19.</li> </ul>
MA.K.NSO.2.3	<p>Locate, order and compare numbers from 0 to 20 using the number line and terms less than, equal to or greater than.</p> <p><b>Access Point</b></p> <p>MA.K.NSO.2.AP.3 Locate and compare two numbers from 0 to 10 to determine which number is less than, equal to or greater than the other number.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of “greater than” as more objects, “less than” as fewer objects, and “equal to” as the same number of objects.</li> <li>• Use objects to represent given numbers from 0 to 10.</li> <li>• Use 1-to-1 matching of objects to determine which number represents a group that has more (is greater than) or fewer (is less than), or if the numbers represent groups that have the same number of objects (are equal).</li> <li>• Express number names (rote count) from 1-10.</li> <li>• Understand the concept of “greater than” as a higher number, “less than” as a lower number, and “equal to” as the same number.</li> </ul>
<p><b><i>MA.K.NSO.3 Develop an understanding of addition and subtraction operations with one-digit whole numbers.</i></b></p>	
MA.K.NSO.3.1	Explore addition of two whole numbers from 0 to 10, and

	related subtraction facts.
	<b>Access Point</b> MA.K.NSO.3.AP.1 Explore addition and subtraction of two whole numbers within 5 using objects.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Given a real-world context use objects to represent the actions “add to” or “take from”.</li> </ul>
MA.K.NSO.3.2	Add two one-digit whole numbers with sums from 0 to 10 and subtract using related facts with procedural reliability.
	<b>Access Point</b> MA.K.NSO.3.AP.2 Apply a strategy for adding and subtracting two one-digit whole numbers to solve within 5.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Given a real-world context use objects to represent the actions “add to” or “take from”.</li> <li>Given an addition or subtraction expression (e.g., <math>2 + 3</math>; <math>4 - 1</math>), use objects to represent the expression.</li> </ul>

### Algebraic Reasoning

<b><i>MA.K.AR.1 Represent and solve addition problems with sums between 0 and 10 and subtraction problems using related facts.</i></b>	
MA.K.AR.1.1	For any number from 1 to 9, find the number that makes 10 when added to the given number.
	<b>Access Point</b> MA.K.AR.1.AP.1 For any number from 1 to 9, use objects to find the number that makes 10 when added to the given number.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Recognize that a full 10-frame represents the number 10.</li> <li>Use 1-to-1 correspondence to count up to 10 objects.</li> <li>Understand addition as “adding to”.</li> <li>Discriminate between the group of objects being “added to” and the group of objects being added.</li> </ul>

<p>MA.K.AR.1.2</p>	<p>Given a number from 0 to 10, find the different ways it can be represented as the sum of two numbers.</p> <p><b>Access Point</b></p> <p>MA.K.AR.1.AP.2 Given a number from 0 to 5, find the different ways it can be represented as the sum of two numbers.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use 1-to-1 correspondence to count up to 5.</li> <li>• Discriminate between the whole and the parts.</li> </ul>
<p>MA.K.AR.1.3</p>	<p>Solve addition and subtraction real-world problems using objects, drawings or equations to represent the problem.</p> <p><b>Access Point</b></p> <p>MA.K.AR.1.AP.3 Solve addition and subtraction real-world problems within 5 using objects, drawings or equations to represent the problem.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” with objects.</li> <li>• Add or subtract within 5.</li> </ul>
<p><b><i>MA.K.AR.2 Develop an understanding of the equal sign.</i></b></p>	
<p>MA.K.AR.2.1</p>	<p>Explain why addition or subtraction equations are true using objects or drawings.</p> <p><b>Access Point</b></p> <p>MA.K.AR.2.AP.1 Show that an addition or subtraction equation within 5 is true using objects or drawings.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand addition as “adding to” and subtraction as “taking from”.</li> <li>• Understand the concept of “equality” as the balance of two values (e.g., a balance scale is level if the values are equal) and understand that if the values on either side of the equal sign are the same, then the equation is true.</li> <li>• Understand that = is “equal to”.</li> <li>• Given an addition or subtraction equation (e.g., <math>2 + 3 = 5</math>; <math>4 - 1 = 3</math>), use objects or drawings to represent the addition or</li> </ul>

subtraction within 5.

## Measurement

<b><i>MA.K.M.1 Identify and compare measurable attributes of objects.</i></b>	
MA.K.M.1.1	Identify the attributes of a single object that can be measured such as length, volume or weight.
	<b>Access Point</b> MA.K.M.1.AP.1 Explore the attributes of a single object that can be measured such as length or weight.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"><li>• Recognize the difference between length and weight.</li></ul>
MA.K.M.1.2	Directly compare two objects that have an attribute which can be measured in common. Express the comparison using language to describe the difference.
	<b>Access Point</b> MA.K.M.1.AP.2 Directly compare two objects to determine which is longer/shorter or heavier/lighter.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"><li>• Recognize the difference between length and weight and how each is measured.</li><li>• Understand that length can be described as longer or shorter.</li><li>• Understand that weight can be described as heavier or lighter.</li></ul>
MA.K.M.1.3	Express the length of an object, up to 20 units long, as a whole number of lengths by laying non-standard objects end to end with no gaps or overlaps.
	<b>Access Point</b> MA.K.M.1.AP.3 Express the length of an object, up to 10 units long, as a whole number of lengths using non-standard objects laid end to end with no gaps or overlaps.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"><li>• Understand that length is an attribute of objects that can be measured.</li><li>• Identify the beginning and end point of the object that needs to be measured.</li></ul>

- Understand that the length measurement of an object is the total number of same sized length units.

## Geometric Reasoning

### ***MA.K.GR.1 Identify, compare and compose two- and three-dimensional figures.***

MA.K.GR.1.1	<p>Identify two- and three-dimensional figures regardless of their size or orientation. Figures are limited to circles, triangles, rectangles, squares, spheres, cubes, cones and cylinders.</p> <p><b>Access Point</b></p> <p>MA.K.GR.1.AP.1 Identify two- and three-dimensional figures regardless of their size. Figures are limited to circles, triangles, rectangles, squares, spheres, cubes, cones and cylinders.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize the defining attributes of circles, triangles, rectangles, squares, spheres, cubes, cones, and cylinders</li> </ul>
MA.K.GR.1.2	<p>Compare two-dimensional figures based on their similarities, differences and positions. Sort two-dimensional figures based on their similarities and differences. Figures are limited to circles, triangles, rectangles and squares.</p> <p><b>Access Point</b></p> <p>MA.K.GR.1.AP.2a Sort two-dimensional figures based on their similarities. Figures are limited to circles, triangles, rectangles and squares.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of “same”.</li> <li>• Understand objects can be sorted by various attributes.</li> </ul> <p>MA.K.GR.1.AP.2b Use informal spatial language to describe the relative positions of two-dimensional figures (e.g., above, below, beside, next to, under).</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize the presence of two separate</li> </ul>



	<p>figures in a given field.</p> <ul style="list-style-type: none"> <li>• Understand the relationship between objects can be described using their position.</li> </ul>
MA.K.GR.1.3	<p>Compare three-dimensional figures based on their similarities, differences and positions. Sort three-dimensional figures based on their similarities and differences. Figures are limited to spheres, cubes, cones and cylinders.</p> <p><b>Access Point</b></p> <p>MA.K.GR.1.AP.3a Sort three-dimensional figures based on their similarities. Figures are limited to spheres, cubes, cones and cylinders.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand concept of “same”.</li> <li>• Understand objects can be sorted by various attributes.</li> </ul> <p>MA.K.GR.1.AP.3b Use informal spatial language to describe the relative positions of three-dimensional figures (e.g., above, below, beside, next to, under).</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize the presence of two separate figures in a given field.</li> <li>• Understand the relationship between objects can be described using their position.</li> </ul>
MA.K.GR.1.4	<p>Find real-world objects that can be modeled by a given two- or three-dimensional figure. Figures are limited to circles, triangles, rectangles, squares, spheres, cubes, cones and cylinders.</p> <p><b>Access Point</b></p> <p>MA.K.GR.1.AP.4 Explore real-world objects that can be modeled by a given two- or three-dimensional figure. Figures are limited to circles, triangles, rectangles, squares, spheres, cubes, cones and cylinders.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize the defining attributes of circles, triangle, rectangles, squares, spheres, cubes, cones and cylinders.</li> </ul>
MA.K.GR.1.5	<p>Combine two-dimensional figures to form a given</p>

	composite figure. Figures used to form a composite shape are limited to triangles, rectangles and squares.
	<p><b>Access Point</b></p> <p>MA.K.GR.1.AP.5 Recognize that a different figure can be formed by combining two smaller two-dimensional figures. Figures used to form a composite shape are limited to triangles, rectangles and squares.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Differentiate between “smaller” and “larger” figures.</li> </ul>

### **Data Analysis and Probability**

<b><i>MA.K.DP.1 Develop an understanding for collecting, representing and comparing data.</i></b>	
MA.K.DP.1.1	Collect and sort objects into categories and compare the categories by counting the objects in each category. Report the results verbally, with a written numeral or with drawings.
	<p><b>Access Point</b></p> <p>MA.K.DP.1.AP.1 Sort objects by characteristic (e.g., size, shape or color). Count the objects in each category and report the results.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of “same”.</li> <li>• Recognize similarities in size, shape, or color.</li> <li>• Use 1:1 correspondence to count up to 10 objects.</li> <li>• Demonstrate that counting has cardinality in the numbers (last number named when counting tells the number of objects counted).</li> </ul>

### **Grade 1 B.E.S.T. Standards Access Points Number Sense and Operations**

<b><i>MA.1.NSO.1 Extend counting sequences and understand the place value of two-digit numbers.</i></b>	
MA.1.NSO.1.1	Starting at a given number, count forward and backwards within 120 by ones. Skip count by 2s to 20 and by 5s to

	<p>100.</p> <p><b>Access Point</b></p> <p>MA.1.NSO.1.AP.1 Starting at a given number, count forward within 100 and backwards within 20 by ones. Skip count by 5s from 5 to 100.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand there is a consistent order when counting.</li> <li>• Understand the concepts of “forward” and “backward”.</li> <li>• Express number names from 1 to 100 by ones.</li> </ul>
MA.1.NSO.1.2	<p>Read numbers from 0 to 100 written in standard form, expanded form and word form. Write numbers from 0 to 100 using standard form and expanded form.</p> <p><b>Access Point</b></p> <p>MA.1.NSO.1.AP.2 Read numbers from 0 to 20 written in standard form and expanded form. Generate numbers from 0 to 20 using standard form.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Express number names (rote count) up to 20.</li> <li>• Identify a number written in standard form when given the name of the number up to 20.</li> <li>• Recognize the numbers from 11-19 and can be represented as one group of 10 ones plus some further ones (expanded form).</li> </ul>
MA.1.NSO.1.3	<p>Compose and decompose two-digit numbers in multiple ways using tens and ones. Demonstrate each composition or decomposition with objects, drawings, and expressions or equations.</p> <p><b>Access Point</b></p> <p>MA.1.NSO.1.AP.3 Compose and decompose numbers up to 20 using tens and ones. Demonstrate each composition or decomposition with objects, drawings, and expressions or equations.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize the numbers from 11-19 and can be represented as one group of 10 ones plus some further ones.</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod).</li> <li>• Count on from 1 ten up to 20.</li> <li>• Given a two-digit number up to 20, understand that the digit in the tens place represents the number of tens and the digit in the ones place represents the further ones.</li> </ul>
MA.1.NSO.1.4	<p>Plot, order and compare whole numbers up to 100.</p> <p><b>Access Point</b> MA.1.NSO.1.AP.4 Order (e.g., 5, 9, 13) and compare (e.g., <math>11 &lt; 19</math>) whole numbers up to 20.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of “greater than” as more objects, “less than” as fewer objects, and “equal to” as the same number of objects.</li> <li>• Use objects to represent given numbers from 0 to 20.</li> <li>• Use 1-to-1 matching of objects to determine which number represents a group that has more (is greater than) or fewer (is less than), or if the numbers represent groups that have the same number of objects (are equal).</li> <li>• Express number names (rote count) from 1-20.</li> <li>• Understand the concept of “greater than” as a higher number, “less than” as a lower number, and “equal to” as the same number.</li> <li>• Understand that <math>&gt;</math> is “greater than”, <math>&lt;</math> is “less than”, and <math>=</math> is “equal to”.</li> </ul>
<b>MA.1.NSO.2 Develop an understanding of addition and subtraction operations with one- and two-digit numbers.</b>	
MA.1.NSO.2.1	<p>Recall addition facts with sums to 10 and related subtraction facts with automaticity.</p> <p><b>Access Point</b> MA.1.NSO.2.AP.1 Recall addition facts with sums to 5 and related subtraction facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given an addition or subtraction expression</li> </ul>

	(e.g., $2 + 3$ ; $4 - 1$ ), use objects or strategies to solve within 5.
MA.1.NSO.2.2	Add two whole numbers with sums from 0 to 20 and subtract using related facts with procedural reliability.
	<b>Access Point</b> MA.1.NSO.2.AP.2 Apply a strategy for adding and subtracting two one-digit whole numbers to solve within 10.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Given a real-world context use objects to represent the actions “add to” or “take from”.</li> <li>Given an addition or subtraction expression (e.g., <math>3 + 4</math>; <math>8 - 1</math>), use objects to represent the expression.</li> </ul>
MA.1.NSO.2.3	Identify the number that is one more, one less, ten more and ten less than a given two-digit number.
	<b>Access Point</b> MA.1.NSO.2.AP.3 Identify the number that is one more and one less than a given number within 20.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Count forward and backward within 20 by ones from any given number.</li> <li>Understand that “one more” is the next counting number and “one less” is the previous counting number.</li> </ul>
MA.1.NSO.2.4	Explore the addition of a two-digit number and a one-digit number with sums to 100.
	<b>Access Point</b> MA.1.NSO.2.AP.4 Explore the addition of a two-digit number from 11 to 19 and a one-digit number.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Understand that the digit in the tens place represents the number of tens and the digit in the ones place represents the number of ones.</li> <li>Use objects (e.g., ten-rods and unit cubes) to represent teen numbers as tens and ones.</li> <li>Understand that addition is “adding to”.</li> <li>Recognize the numbers from 11-19 and can</li> </ul>

	<p>be represented as one group of 10 ones plus some further ones.</p> <ul style="list-style-type: none"> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod).</li> </ul>
MA.1.NSO.2.5	<p>Explore subtraction of a one-digit number from a two-digit number.</p> <p><b>Access Point</b></p> <p>MA.1.NSO.2.AP.5 Explore subtraction of a one-digit number from a two-digit number from 11 to 19.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that the digit in the tens place represents the number of tens and the digit in the ones place represents the number of ones.</li> <li>• Use objects (e.g., ten-rods and unit cubes) to represent teen numbers as tens and ones.</li> <li>• Understand that subtraction is “take from”.</li> <li>• Understand that 1 ten is equal to a group of 10 ones (e.g., 1 ten-rod is equal to 10-unit cubes).</li> </ul>

### Fractions

<b><i>MA.1.FR.1 Develop an understanding of fractions by partitioning shapes into halves and fourths.</i></b>	
MA.1.FR.1.1	<p>Partition circles and rectangles into two and four equal-sized parts. Name the parts of the whole using appropriate language including halves or fourths.</p> <p><b>Access Point</b></p> <p>MA.1.FR.1.AP.1 Partition circles and rectangles into two and four equal-sized parts. Recognize the parts of the whole as halves or fourths.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize if parts have equal sizes.</li> <li>• Recognize that a larger figure can be formed by combining smaller two-dimensional figures.</li> </ul>

### Algebraic Reasoning

<b><i>MA.1.AR.1 Solve addition problems with sums between 0 and 20 and subtraction problems using related facts.</i></b>	
MA.1.AR.1.1	Apply properties of addition to find a sum of three or more whole numbers.
	<b>Access Point</b> MA.1.AR.1.AP.1 Apply the commutative property of addition to find a sum of two whole numbers within 20.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Represent addition expressions using objects to find sums.</li> <li>• Recognize that when given an addition expression that changing the order of the addends does not change the sum.</li> <li>• Recognize the greater addend in an addition expression.</li> <li>• Count on from a given number within 20.</li> </ul>
MA.1.AR.1.2	Solve addition and subtraction real-world problems using objects, drawings or equations to represent the problem.
	<b>Access Point</b> MA.1.AR.1.AP.2 Solve addition and subtraction real-world problems within 10 using objects, drawings or equations to represent the problem.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” with objects.</li> <li>• Add or subtract within 10.</li> </ul>
<b><i>MA.1.AR.2 Develop an understanding of the relationship between addition and subtraction.</i></b>	
MA.1.AR.2.1	Restate a subtraction problem as a missing addend problem using the relationship between addition and subtraction.
	<b>Access Point</b> MA.1.AR.2.AP.1 Use the relationship between addition and subtraction to explore subtraction as addition with a missing addend.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Model addition and subtraction expressions</li> </ul>

	<p>with objects.</p> <ul style="list-style-type: none"> <li>Given an addition or subtraction expression (e.g., <math>3 + 4</math>; <math>8 - 1</math>), use objects to solve within 10.</li> </ul>
MA.1.AR.2.2	<p>Determine and explain if equations involving addition or subtraction are true or false.</p> <p><b>Access Point</b></p> <p>MA.1.AR.2.AP.2 Determine if addition or subtraction equations (with no more than three terms) are true or false. Sums may not exceed 10 and their related subtraction facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Use objects to find sums within 10 and their related subtraction facts.</li> <li>Understand the concept of “equality” as the balance of two values (e.g., if a balance scale is level, then the values are equal and if it is not level, then the values are not equal).</li> <li>Understand that <math>=</math> is “equal to”.</li> <li>Understand that if the values on either side of the equal sign are the same, then the equation is true and if the values on either side of the equal side are not the same, then the equation is false.</li> </ul>
MA.1.AR.2.3	<p>Determine the unknown whole number in an addition or subtraction equation, relating three whole numbers, with the unknown in any position.</p> <p><b>Access Point</b></p> <p>MA.1.AR.2.AP.3 Determine the unknown whole number in an addition or subtraction equation, relating three whole numbers, with the result unknown (e.g., <math>8 - 2 = \underline{\quad}</math>, <math>\underline{\quad} = 7 + 3</math>). Sums may not exceed 10 and their related subtraction facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Given an addition or subtraction expression (e.g., <math>8 - 2</math>; <math>7 + 3</math>) use objects to solve within 10.</li> <li>Understand a symbol (e.g., <math>\underline{\quad}</math> or <math>\square</math>) may be used to represent an unknown sum or difference in an equation.</li> </ul>



- Understand that = is “equal to”.

## Measurement

### ***MA.1.M.1 Compare and measure the length of objects.***

MA.1.M.1.1

Estimate the length of an object to the nearest inch. Measure the length of an object to the nearest inch or centimeter.

**Access Point**

MA.1.M.1.AP.1.a Use a ruler to measure the length of an object with exact whole units to the nearest inch.

**Essential Understandings:**

- Understand that length is an attribute of objects that can be measured.
- Identify the beginning and end point of the object that needs to be measured.
- Recognize that the units marked on a ruler have equal length intervals. Understand that the total number of equal interval distances, spanned end to end, can be counted to determine the overall length of an object.

MA.1.M.1.AP.1.b Explore familiar objects that can be used to develop a mental measurement benchmark to understand the relative size of an inch.

**Essential Understandings:**

- Understand that length is an attribute of objects that can be measured.
- Understand that length is an attribute that can be measured in inches.
- Recognize that the units marked on an inch ruler have equal length intervals and that each one of these length intervals represents the length of 1 inch.
- Use a ruler to measure the length of objects that are exactly 1 inch long.
- Compare the length of up to three objects, each measuring 1 inch, using direct comparison and recognize that they are all the same length.

MA.1.M.1.2

Compare and order the length of up to three objects using direct and indirect comparison.

	<p><b>Access Point</b>  MA.1.M.1.AP.2 Compare and order the length of up to three objects using direct comparison.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that length is an attribute that can be measured.</li> <li>• Understand that length can be described as longer/longest, shorter/shortest in relation to other objects.</li> <li>• Understand that beginning points of each objects' length must be aligned in order to directly compare the overall length of the objects.</li> </ul>
<p><b><i>MA.1.M.2 Tell time and identify the value of coins and combinations of coins and dollar bills.</i></b></p>	
<p>MA.1.M.2.1</p>	<p>Using analog and digital clocks, tell and write time in hours and half-hours.</p> <p><b>Access Point</b>  MA.1.M.2.AP.1 Using analog and digital clocks, express the time in hours.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that time is an attribute that can be measured with a clock and can be expressed in hours.</li> <li>• Recognize numerals 1-12.</li> </ul>
<p>MA.1.M.2.2</p>	<p>Identify pennies, nickels, dimes and quarters, and express their values using the ¢ symbol. State how many of each coin equal a dollar.</p> <p><b>Access Point</b>  MA.1.M.2.AP.2 Identify the names and values of pennies, nickels, dimes, and quarters.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that coins (pennies, nickels, dimes, and quarters) are a type of currency.</li> <li>• Understand that coins can be offered in exchange for goods and services.</li> <li>• Sort coins by size and color</li> </ul>

MA.1.M.2.3	Find the value of combinations of pennies, nickels and dimes up to one dollar, and the value of combinations of one-, five- and ten-dollar bills up to \$100. Use the ¢ and \$ symbols appropriately.
	<b>Access Point</b> MA.1.M.2.AP.3a Find the value of a group of only pennies, only nickels or only dimes up to \$1.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Identify the values of pennies, nickels, and dimes.</li> <li>• Count by 1's up to 100, skip count by 5's up to 100, and skip count by 10's up to 100.</li> </ul>
	MA.1.M.2.AP.3b Find the value of a group of only one-, only five- or only ten-dollar bills up to \$100.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Identify the values of one-, five-, and ten-dollar bills.</li> <li>• Count by 1's up to 100, skip count by 5's up to 100, and skip count by 10's up to 100.</li> </ul>

### Geometric Reasoning

<b><i>MA.1.GR.1 Identify and analyze two- and three-dimensional figures based on their defining attributes.</i></b>	
MA.1.GR.1.1	Identify, compare and sort two- and three-dimensional figures based on their defining attributes. Figures are limited to circles, semi-circles, triangles, rectangles, squares, trapezoids, hexagons, spheres, cubes, rectangular prisms, cones and cylinders.
	<b>Access Point</b> MA.1.GR.1.AP.1 Sort and identify two- or three-dimensional figures based on their defining attributes. (e.g., number of sides, vertices, edges, faces, etc., rather than color, orientation, or size). Figures are limited to circles, semi-circles, triangles, rectangles, squares, trapezoids, hexagons, spheres, cubes, rectangular prisms, cones and cylinders.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand concept of "same".</li> <li>• Understand objects can be sorted by various</li> </ul>

	<p>attributes.</p> <ul style="list-style-type: none"> <li>Identify specified defining attributes (i.e., sides, vertices, edges, faces) in isolated two- or three-dimensional figures</li> </ul>
MA.1.GR.1.2	<p>Sketch two-dimensional figures when given defining attributes. Figures are limited to triangles, rectangles, squares, and hexagons.</p>
	<p><b>Access Point</b></p> <p>MA.1.GR.1.AP.2 Produce two-dimensional figures when given defining attributes. Figures are limited to triangles, rectangles and squares.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Identify specified defining attributes (i.e., sides, vertices, closed versus open) in isolated two-dimensional figures</li> </ul>
MA.1.GR.1.3	<p>Compose and decompose two- and three-dimensional figures. Figures are limited to semi-circles, triangles, rectangles, squares, trapezoids, hexagons, cubes, rectangular prisms, cones, and cylinders.</p>
	<p><b>Access Point</b></p> <p>MA.1.GR.1.AP.3 Recognize that different figures can be formed by putting together smaller two- or three-dimensional figures and that smaller figures can be formed by taking apart larger two- or three-dimensional figures. Figures are limited to semi-circles, triangles, rectangles, squares, trapezoids, hexagons, cubes, rectangular prisms, cones, and cylinders.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Recognize that a larger figure can be formed by combining two smaller two-dimensional figures</li> </ul>
MA.1.GR.1.4	<p>Given a real-world object, identify parts that are modeled by two- and three-dimensional figures. Figures are limited to semi-circles, triangles, rectangles, squares and hexagons, spheres, cubes, rectangular prisms, cones and cylinders.</p>
	<p><b>Access Point</b></p> <p>MA.1.GR.1.AP.4 Explore real-world objects with parts that can be modeled by a given two- or three-dimensional figure. Figures are limited to semi-circles, triangles,</p>

rectangles, squares and hexagons, spheres, cubes, rectangular prisms, cones, and cylinders.

**Essential Understandings:**

- Recognize the defining attributes of semi-circles, triangles, rectangles, squares and hexagons, spheres, cubes, rectangular prisms, cones and cylinders.

**Data Analysis and Probability**

***MA.1.DP.1 Collect, represent and interpret data using pictographs and tally marks.***

MA.1.DP.1.1

Collect data into categories and represent the results using tally marks or pictographs.

**Access Point**

MA.1.DP.1.AP.1 Sort data into two categories and represent the results using tally marks or pictographs.

**Essential Understandings:**

- Understand the concept of “same”.
- Use 1-to-1 correspondence.

MA.1.DP.1.2

Interpret data represented with tally marks or pictographs by calculating the total number of data points and comparing the totals of different categories.

**Access Point**

MA.1.DP.1.AP.2 Interpret data represented with tally marks or pictographs to determine how many in each category and compare the values of two categories of data in terms of more or less.

**Essential Understandings:**

- Understand that each category represents a group with a characteristic in common.
- Understand that each tally mark or picture represents one data point from that category.
- Understand that the total number of tally marks or pictures in each category tells “how many” in each category.
- Understand the concept of “more” and “less”.

## Number Sense and Operations

<b><i>MA.2.NSO.1 Understand the place value of three-digit numbers.</i></b>	
MA.2.NSO.1.1	<p>Read and write numbers from 0 to 1,000 using standard form, expanded form and word form.</p> <p><b>Access Point</b></p> <p>MA.2.NSO.1.AP.1 Read and generate numbers from 0 to 100 using standard form and expanded form.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Express number names (rote count) up to 100.</li> <li>• Identify a number written in standard form when given the name of the number up to 100.</li> <li>• Understand that the 3 digits of a three-digit number represent an amount of hundreds, tens, and ones.</li> <li>• Understand that expanded form is the value of the hundreds, plus the value of the tens plus the value of the ones.</li> <li>• Understand that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to 1, 2, 3, 4, 5, 6, 7, 8, 9 tens and 0 further ones.</li> <li>• Generate numbers 0-9 using standard form.</li> </ul>
MA.2.NSO.1.2	<p>Compose and decompose three-digit numbers in multiple ways using hundreds, tens and ones. Demonstrate each composition or decomposition with objects, drawings, and expressions or equations.</p> <p><b>Access Point</b></p> <p>MA.2.NSO.1.AP.2 Compose and decompose two-digit numbers using tens and ones. Demonstrate each composition or decomposition with objects, drawings, expressions or equations.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod). and a group of 10 tens is equal to 1 hundred (e.g., 10 ten-rods is equal to 1 hundred flat).</li> <li>• Given a number up to 100, understand that</li> </ul>

	<p>the digit in the hundreds place represents the number of hundreds, the digit in the tens place represents the number of tens and the digit in the ones place represents the number of ones.</p> <ul style="list-style-type: none"> <li>• Use objects to represent numbers up to 100 using hundreds, tens, and ones.</li> <li>• Skip count by 10's and count on from decade numbers count by ones.</li> </ul>
<p>MA.2.NSO.1.3</p>	<p>Plot, order and compare whole numbers up to 1,000.</p> <p><b>Access Point</b> MA.2.NSO.1.AP.3 Plot, order and compare whole numbers up to 100.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a number up to 100, understand that the digit in the hundreds place represents the number of hundreds, the digit in the tens place represents the number of tens, and the digit in the ones place represents the number of ones.</li> <li>• Use objects to represent numbers up to 100 using hundreds, tens, and ones.</li> <li>• Use matching of same unit objects (flats, rods, cubes) to compare starting with the hundreds place.</li> <li>• Understand that <math>&gt;</math> is "greater than", <math>&lt;</math> is "less than", and <math>=</math> is "equal to".</li> <li>• Understand that numbers on a number line are plotted in sequential order and numbers that are lower on the number line have a lesser value and numbers higher on the number line have a greater value.</li> </ul>
<p>MA.2.NSO.1.4</p>	<p>Round whole numbers from 0 to 100 to the nearest 10.</p> <p><b>Access Point</b> MA.2.NSO.1.AP.4 Round whole numbers from 0 to 100 to the nearest 10 with visual support.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a number up to 100, understand that the digit in the hundreds place represents the</li> </ul>

	<p>number of hundreds, the digit in the tens place represents the number of tens, and the digit in the ones place represents the number of ones.</p> <ul style="list-style-type: none"> <li>• Understand that numbers on a number line are plotted in sequential order, numbers that are lower on the number line have a lesser value, and numbers higher on the number line have a greater value.</li> <li>• Plot whole numbers up to 100 on a number line.</li> <li>• Identify which decade the number being rounded is closest to on a number line.</li> </ul>
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**MA.2.NSO.2 Add and subtract two- and three-digit whole numbers.**

MA.2.NSO.2.1	<p>Recall addition facts with sums to 20 and related subtraction facts with automaticity.</p> <p><b>Access Point</b></p> <p>MA.2.NSO.2.AP.1 Recall addition facts with sums to 10 and related subtraction facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given an addition or subtraction expression (e.g., <math>3 + 4</math>; <math>8 - 1</math>), use objects or strategies to solve within 10.</li> </ul>
MA.2.NSO.2.2	<p>Identify the number that is ten more, ten less, one hundred more and one hundred less than a given three-digit number.</p> <p><b>Access Point</b></p> <p>MA.2.NSO.2.AP.2 Identify the number that is ten more or ten less than a given two-digit number.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that the digit in the tens place represents the number of tens and the digit in the ones place represents the number of ones.</li> <li>• Use objects (e.g., ten-rods and unit cubes) to represent numbers up to 99 using tens and ones.</li> <li>• Understand that “ten more” increases the number of tens by 1 ten and that “ten less” decreases the number of tens by 1 ten.</li> </ul>



	<ul style="list-style-type: none"> <li>• Understand that a group of 10 tens is equal to 1 hundred (e.g., 10 ten-rods is equal to 1 hundred flat).</li> </ul>
MA.2.NSO.2.3	<p>Add two whole numbers with sums up to 100 with procedural reliability. Subtract a whole number from a whole number, each no larger than 100, with procedural reliability.</p> <p><b>Access Point</b> MA.2.NSO.2.AP.3 Apply a strategy for adding and subtracting a two-digit number (from 11 to 19) and a single digit whole number.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that the digit in the tens place represents the number of tens and the digit in the ones place represents the number of ones.</li> <li>• Use objects (e.g., ten-rods and unit cubes) to represent teen numbers as tens and ones.</li> <li>• Understand that addition is “adding to” and that subtraction is “take from”.</li> <li>• Understand that a group of 10 ones is equal to 1 ten and 1 ten is equal to a group of 10 ones.</li> <li>• Understand that in adding it is sometimes necessary to compose a ten and in subtracting it is sometimes necessary to decompose a ten.</li> <li>• Understand that in adding two-digit numbers one adds tens and tens and ones and ones and sometimes it is necessary to compose a ten.</li> </ul>
MA.2.NSO.2.4	<p>Explore the addition of two whole numbers with sums up to 1,000. Explore the subtraction of a whole number from a whole number, each no larger than 1,000.</p> <p><b>Access Point</b> MA.2.NSO.2.AP.4 Explore the addition of a two-digit and a single-digit whole number with sums up to 100. Explore the subtraction of a one-digit from a two-digit whole number.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that the digit in the hundreds place represents the number of hundreds, the</li> </ul>

	<p>digit in the tens place represents the number of tens and the digit in the ones place represents the number of ones.</p> <ul style="list-style-type: none"> <li>• Use objects (e.g., ten-rods and unit cubes) to represent two-digit numbers as tens and ones.</li> <li>• Understand that addition is “adding to” and subtraction is “take from”.</li> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod) and that 1 ten is equal to a group of 10 ones (e.g., 1 ten-rod is equal to 10-unit cubes)</li> <li>• Understand that in adding it is sometimes necessary to compose a ten and in subtracting it is sometimes necessary to decompose a ten.</li> </ul>
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**Fractions**

<b><i>MA.2.FR.1 Develop an understanding of fractions.</i></b>	
MA.2.FR.1.1	Partition circles and rectangles into two, three or four equal-sized parts. Name the parts using appropriate language, and describe the whole as two halves, three thirds or four fourths.
	<p><b>Access Point</b></p> <p>MA.2.FR.1.AP.1 Partition circles and rectangles into two, three or four equal-sized parts. Recognize the parts of the whole as halves, thirds or fourths. Explore the whole as two halves, three thirds or four fourths.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize if parts have equal sizes.</li> <li>• Recognize that a larger figure can be formed by combining smaller two-dimensional figures.</li> </ul>
MA.2.FR.1.2	Partition rectangles into two, three or four equal-sized parts in two different ways showing that equal-sized parts of the same whole may have different shapes.
	<p><b>Access Point</b></p> <p>MA.2.FR.1.AP.2 Partition rectangles into two or four equal-sized parts in two different ways showing that equal-sized parts of the same whole may have different shapes.</p>

**Essential Understandings:**

- Recognize if parts have equal sizes.
- Recognize that a larger figure can be formed by combining smaller two-dimensional figures.

**Algebraic Reasoning*****MA.2.AR.1 Solve addition problems with sums between 0 and 100 and related subtraction problems.***

MA.2.AR.1.1

Solve one- and two-step addition and subtraction real-world problems.

**Access Point**

MA.2.AR.1.AP.1 Solve one-step addition and subtraction real-world problems within 20 using objects.

**Essential Understandings:**

- Represent addition and subtraction situations involving “adding to” and “taking from” with objects or drawings.
- Add or subtract within 20.

***MA.2.AR.2 Demonstrate an understanding of equality and addition and subtraction.***

MA.2.AR.2.1

Determine and explain whether equations involving addition and subtraction are true or false.

**Access Point**

MA.2.AR.2.AP.1 Determine if addition or subtraction equations with no more than three terms are true or false. Sums may not exceed 20 and their related subtraction facts.

**Essential Understandings:**

- Use objects to find sums within 20 and their related subtraction facts.
- Understand the concept of “equality” as the balance of two values (e.g., if a balance scale is level, then the values are equal and if it is not level, then the values are not equal).
- Understand that = is “equal to”.
- Understand that if the values on either side of the equal sign are the same, then the equation is true and if the values on either

	side of the equal side are not the same, then the equation is false.
MA.2.AR.2.2	Determine the unknown whole number in an addition or subtraction equation, relating three or four whole numbers, with the unknown in any position.
	<b>Access Point</b> MA.2.AR.2.AP.2 Determine the unknown whole number in an addition or subtraction equation, relating three whole numbers, with the change or result unknown (e.g., $7 + \_ = 10$ , $10 - 3 = \blacksquare$ ). Sums may not exceed 20 and their related subtraction facts.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Given an addition or subtraction expression (e.g., <math>8 - 2</math>; <math>7 + 3</math>) use objects to solve within 20.</li> <li>Understand a symbol (e.g., <math>\_</math> or <math>\square</math>) may be used to represent an unknown number in an equation.</li> <li>Understand that <math>=</math> is "equal to".</li> </ul>
<b>MA.2.AR.3 Develop an understanding of multiplication.</b>	
MA.2.AR.3.1	Represent an even number using two equal groups or two equal addends. Represent an odd number using two equal groups with one left over or two equal addends plus 1.
	<b>Access Point</b> MA.2.AR.3.AP.1 Explore the concept of odd and even by pairing objects to represent an even number using two equal groups or represent an odd number by using two equal groups with one left over. Group of objects may not exceed 20.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Use 1-to-1 correspondence to pair objects.</li> </ul>
MA.2.AR.3.2	Use repeated addition to find the total number of objects in a collection of equal groups. Represent the total number of objects using rectangular arrays and equations.
	<b>Access Point</b> MA.2.AR.3.AP.2 Explore using repeated addition to find the total number of objects represented in a collection of equal groups (e.g., 3 groups of 2 objects) or in a

rectangular array (e.g., 3 rows of 2 objects). Total objects may not exceed 20.

**Essential Understandings:**

- Understand the concept of equal groups.
- Distinguish between the number of groups and the number in each group.
- Understand the concept of a rectangular array.
- Distinguish between the number of rows and the number in each row.
- When given up to 20 objects, organized in equal groups or in a rectangular array, use 1:1 correspondence to find the total number of objects.
- When given up to 20 objects, organized in equal groups or in a rectangular array, recognize that the number in each group/row is the same.

**Measurement**

***MA.2.M.1 Measure the length of objects and solve problems involving length.***

MA.2.M.1.1

Estimate and measure the length of an object to the nearest inch, foot, yard, centimeter or meter by selecting and using an appropriate tool.

**Access Point**

MA.2.M.1.AP.1.a Measure the length of an object to the nearest inch, foot and or yard when given the appropriate tool.

**Essential Understandings:**

- Understand that length is an attribute of objects that can be measured using a ruler.
- Identify the beginning and end point of the object that needs to be measured.
- Recognize that the units marked on a ruler/yard stick have equal length intervals.
- Understand that the total number of equal interval distances, spanned end to end, can be counted to determine the overall length of an

	<p>object.</p> <p>MA.2.M.1.AP.1.b Explore estimation strategies by developing measurement benchmarks of familiar objects that could be used to make reasonable estimates of length to the nearest inch, foot, or yard.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that length is an attribute of objects that can be measured.</li> <li>• Understand that length is an attribute that can be measured in inches, feet, or yards.</li> <li>• Recognize that the units marked on a ruler/yard stick have equal length intervals and that each one of these length intervals represents the length of 1 inch, 1 foot, or 1 yard.</li> <li>• Use a ruler/yardstick to measure the length of objects that are exactly 1 inch, 1 foot, or 1 yard long.</li> <li>• Compare the length of up to three objects, all measuring 1 inch, 1 foot, or 1 yard, using direct comparison and recognize that they are all the same length.</li> <li>• Measure the lengths of two objects, both measuring 1 inch, 1 foot, or 1 yard, and recognize when there is no difference between their measurements that both objects are the same length.</li> </ul>
MA.2.M.1.2	<p>Measure the lengths of two objects using the same unit and determine the difference between their measurements.</p> <p><b>Access Point</b></p> <p>MA.2.M.1.AP.2 Measure the lengths of two objects using the same unit (i.e., inch, foot, yard) and determine the difference between their measurements.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that length is an attribute of objects that can be measured using a ruler.</li> <li>• Identify the beginning and end point of the object that needs to be measure.</li> <li>• Recognize that the units marked on a ruler/yard stick have equal length intervals.</li> <li>• Understand that the total number of equal interval distances, spanned end to end, can be</li> </ul>

	counted to determine the overall length of an object.
MA.2.M.1.3	Solve one- and two-step real-world measurement problems involving addition and subtraction of lengths given in the same units.
	<b>Access Point</b> MA.2.M.1.AP.3 Solve one-step real-world measurement problems involving addition and subtraction of lengths within 20 given in the same unit (i.e., inch, foot, yard).
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” length with objects or drawings.</li> <li>• Add or subtract within 20.</li> </ul>
<b>MA.2.M.2 Tell time and solve problems involving money.</b>	
MA.2.M.2.1	Using analog and digital clocks, tell and write time to the nearest five minutes using a.m. and p.m. appropriately. Express portions of an hour using the fractional terms half an hour, half past, quarter of an hour, quarter after and quarter til.
	<b>Access Point</b> MA.2.M.2.AP.1 Using analog and digital clocks, express the time in hours and half hours. Explore the concept of a.m. and p.m.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand that time is an attribute that can be measured with a clock and can be expressed in hours.</li> <li>• Recognize that on an analog clock the longer hand is the minute hand and that the shorter hand is the hour hand.</li> <li>• Recognize that on an analog clock when the longer hand is pointing to 12, and the shorter hand is pointing to one of the numerals 1-12, the numeral being pointed to represents the hour and the time is read as ___ o’clock.</li> <li>• Recognize that on a digital clock the numerals 1-12, before the colon, represent the hours.</li> </ul>

MA.2.M.2.2	Solve one- and two-step addition and subtraction real-world problems involving either dollar bills within \$100 or coins within 100¢ using \$ and ¢ symbols appropriately.
	<p><b>Access Point</b></p> <p>MA.2.M.2.AP.2 Solve one-step addition and subtraction real-world problems involving either dollar bills within \$20 or coins within 20¢. Explore using \$ for dollar bills and ¢ symbol for coins.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” with objects or drawings.</li> <li>• Add or subtract within 20.</li> </ul>

### Geometric Reasoning

<b><i>MA.2.GR.1 Identify and analyze two-dimensional figures and identify lines of symmetry.</i></b>	
MA.2.GR.1.1	Identify and draw two-dimensional figures based on their defining attributes. Figures are limited to triangles, rectangles, squares, pentagons, hexagons and octagons.
	<p><b>Access Point</b></p> <p>MA.2.GR.1.AP.1 Identify and produce two-dimensional figures when given defining attributes. Figures are limited to triangles, rectangles, hexagons and squares.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize the defining attributes of triangles, rectangles, hexagons, and squares.</li> <li>• Identify specified defining attributes (i.e., sides, vertices, closed versus open, straight versus curved) in isolated two-dimensional figures.</li> </ul>
MA.2.GR.1.2	Categorize two-dimensional figures based on the number and length of sides, number of vertices, whether they are closed or not and whether the edges are curved or straight.
	<p><b>Access Point</b></p> <p>MA.2.GR.1.AP.2 Sort two-dimensional figures based on the number of sides, number of vertices, whether they are</p>



	<p>closed or open and whether the sides are curved or straight.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand concept of “same”.</li> <li>• Understand objects can be sorted by various attributes.</li> <li>• Identify specified defining attributes (i.e., sides, vertices, closed versus open, straight versus curved) in isolated two-dimensional figures.</li> </ul>
MA.2.GR.1.3	<p>Identify line(s) of symmetry for a two-dimensional figure.</p> <p><b>Access Point</b></p> <p>MA.2.GR.1.AP.3 Identify a line of symmetry for a two-dimensional figure.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize equal parts.</li> </ul>
<b><i>MA.2.GR.2 Describe perimeter and find the perimeter of polygons.</i></b>	
MA.2.GR.2.1	<p>Explore perimeter as an attribute of a figure by placing unit segments along the boundary without gaps or overlaps. Find perimeters of rectangles by counting unit segments.</p> <p><b>Access Point</b></p> <p>MA.2.GR.2.AP.1 Explore perimeter as an attribute of a figure that can be measured by placing unit segments along the boundary without gaps or overlaps. Find perimeters of rectangles by counting unit segments.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Express the length of an object as a whole number of lengths using non-standard objects laid end to end with no gaps or overlaps.</li> </ul>
MA.2.GR.2.2	<p>Find the perimeter of a polygon with whole-number side lengths. Polygons are limited to triangles, rectangles, squares and pentagons.</p> <p><b>Access Point</b></p> <p>MA.2.GR.2.AP.2 Find the perimeter of a polygon with whole-number side lengths given. Polygons are limited to triangles, rectangles and squares.</p>

**Essential Understandings:**

- Understand that perimeter is the measurement of the total length of the boundary around a figure.
- Add up to 4 single digit whole numbers.

**Data Analysis and Probability**

***MA.2.DP.1 Collect, categorize, represent and interpret data using appropriate titles, labels and units.***

MA.2.DP.1.1

Collect, categorize and represent data using tally marks, tables, pictographs or bar graphs. Use appropriate titles, labels and units.

**Access Point**

MA.2.DP.1.AP.1 Sort data into up to three categories and represent the results using tally marks, tables, pictographs or bar graphs. Align data with given title, labels and units.

**Essential Understandings:**

- Understand that each category represents a group with a characteristic in common.
- Understand that each tally mark or picture represents one data point from that category.
- Understand that the total number of tally marks or pictures in each category tells “how many” in each category.

MA.2.DP.1.2

Interpret data represented with tally marks, tables, pictographs or bar graphs including solving addition and subtraction problems.

**Access Point**

MA.2.DP.1.AP.2 Interpret data represented with tally marks, tables, pictographs or bar graphs to solve one-step put-together and take-apart problems. Pictograph symbols and bar graph intervals may only represent a quantity of 1.

**Essential Understandings:**

- Understand that each category represents a group with a characteristic in common.
- Understand that each tally mark or picture represents one data point from that category.
- Understand that the total number of tally

marks or pictures in each category tells "how many" in each category.

- Understand that the numerals in each section of the table or the height of each bar tells "how many" in each category.
- Understand the terms and location of "title" "labels" and "units".
- Use objects or drawings to represent addition involving "putting together" within 20.
- Use objects or drawings to represent subtraction-involving taking from within 20.

### **Grade 3 B.E.S.T. Standards Access Points Number Sense and Operations**

#### ***MA.3.NSO.1 Understand the place value of four-digit numbers.***

MA.3.NSO.1.1 Read and write numbers from 0 to 10,000 using standard form, expanded form and word form.

##### **Access Point**

MA.3.NSO.1.AP.1 Read and generate numbers from 0 to 1,000 using standard form and expanded form.

##### **Essential Understandings:**

- Express number names (rote count) up to 100.
- Skip count by 100's up to 1,000.
- Identify a number written in standard form when given the name of the number up to 100.
- Understand that the 4 digits of a four-digit number represent an amount of thousands, hundreds, tens, and further ones..
- Understand that expanded form is the value of the thousands, plus the value of the hundreds, plus the value of the tens, plus the value of the ones
- Understand that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to 1, 2, 3, 4, 5, 6, 7, 8, 9 tens and 0 further ones
- Understand that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to 1, 2, 3, 4, 5, 6, 7, 8, 9 hundreds and 0 further tens and 0 further ones.

	<ul style="list-style-type: none"> <li>• Generate numbers 0-100 using standard form.</li> </ul>
MA.3.NSO.1.2	<p>Compose and decompose four-digit numbers in multiple ways using thousands, hundreds, tens and ones. Demonstrate each composition or decomposition using objects, drawings, and expressions or equations.</p> <p><b>Access Point</b></p> <p>MA.3.NSO.1.AP.2 Compose and decompose three-digit numbers using hundreds, tens and ones. Demonstrate each composition or decomposition with objects, drawings, expressions or equations.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that a group of 10 tens is equal to 1 hundred (e.g., 10 ten-rods is equal to 1 hundred flat) and a group of 10 hundreds is equal to 1 thousand (e.g., 10 hundred flats is equal to 1 thousand cube).</li> <li>• Represent numbers up to 1,000 using thousands, hundreds, tens, and ones.</li> <li>• Skip count by 100's.</li> <li>• Count on from century numbers by 10's. Count on from decade numbers by ones.</li> <li>• Given a number up to 1,000, understand that the digit in the thousands place represents the number of thousands, the number in the hundreds place represents the number of hundreds, the number in the tens place represents the number of tens and the digit in the ones place represents the number of ones.</li> </ul>
MA.3.NSO.1.3	<p>Plot, order and compare whole numbers up to 10,000.</p> <p><b>Access Point</b></p> <p>MA.3.NSO.1.AP.3 Plot, order and compare whole numbers up to 1,000.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a number up to 1,000, understand that the digit in the thousands place represents the numbers of thousands, the digit in the hundreds place represents the number of hundreds, the digit in the tens place represents the number of tens, and the digit in the ones place represents the number of ones.</li> </ul>

	<ul style="list-style-type: none"> <li>• Use objects to represent numbers up to 1,000 using thousands, hundreds, tens, and ones.</li> <li>• Use matching of same unit objects (thousands cube, flats, rods, unit cubes) to compare starting with the thousands place.</li> <li>• Understand that <math>&gt;</math> is "greater than", <math>&lt;</math> is "less than", and <math>=</math> is "equal to".</li> <li>• Understand that numbers on a number line are plotted in sequential order, numbers that are lower on the number line have a lesser value, and numbers higher on the number line have a greater value.</li> </ul>
MA.3.NSO.1.4	<p>Round whole numbers from 0 to 1,000 to the nearest 10 or 100.</p> <p><b>Access Point</b></p> <p>MA.3.NSO.1.AP.4 Round whole numbers from 0 to 1,000 to the nearest 100 with visual support.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a number up to 1,000, understand that the digit in the thousands place represents the number of thousands, the digit in the hundreds place represents the number of hundreds, the digit in the tens place represents the number of tens, and the digit in the ones place represents the number of ones.</li> <li>• Understand that numbers on a number line are plotted in sequential order, numbers that are lower on the number line have a lesser value, and numbers higher on the number line have a greater value.</li> <li>• Plot whole numbers up to 1,000 on a number line</li> <li>• Identify which century the number being rounded is closest to on a number line.</li> <li>• Understand that if the number being rounded is halfway between two centuries, then it rounds to the greater century</li> </ul>
<b>MA.3.NSO.2 Add and subtract multi-digit whole numbers. Build an understanding of multiplication and division operations.</b>	
MA.3.NSO.2.1	Add and subtract multi-digit whole numbers including

	<p>using a standard algorithm with procedural fluency.</p> <p><b>Access Point</b></p> <p>MA.3.NSO.2.AP.1 Apply a strategy to add and subtract two two-digit whole numbers.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that the digit in the hundreds place represents the number of hundreds, the digit in the tens place represents the number of tens and the digit in the ones place represents the number of ones.</li> <li>• Use objects (e.g., ten-rods and unit cubes) to represent two-digit numbers as tens and ones.</li> <li>• Understand that addition is “adding to” and subtraction is “take from”.</li> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod) and that 1 ten is equal to a group of 10 ones (e.g., 1 ten-rod is equal to 10-unit cubes).</li> <li>• Understand that a group of 10 tens is equal to 1 hundred (e.g., 10 ten-rods is equal to 1 hundred flat) and that 1 hundred is equal to 10 tens (e.g., 1 hundred flat is equal to 10 ten-rods).</li> <li>• Understand that in adding two-digit numbers one adds tens and tens and ones and ones and sometimes it is necessary to compose a ten and/or a hundred.</li> <li>• Understand that in subtracting two-digit numbers one subtracts tens from tens and ones from ones and sometimes it is necessary to decompose a ten.</li> </ul>
MA.3.NSO.2.2	<p>Explore multiplication of two whole numbers with products from 0 to 144, and related division facts.</p> <p><b>Access Point</b></p> <p>MA.3.NSO.2.AP.2 Explore the concept of multiplication of two single-digit whole numbers using objects.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of equal groups.</li> <li>• Distinguish between the number of groups and the number in each group.</li> <li>• Understand the concept of a rectangular</li> </ul>

	<p>array.</p> <ul style="list-style-type: none"> <li>• Distinguish between the number of rows and the number in each row.</li> <li>• When given up to 20 objects, organized in equal groups or in a rectangular array, use 1:1 correspondence to find the total number of objects.</li> <li>• When given up to 20 objects, organized in equal groups or in a rectangular array, recognize that the number in each group/row is the same and can be repeatedly added to find the total.</li> </ul>
MA.3.NSO.2.3	<p>Multiply a one-digit whole number by a multiple of 10, up to 90, or a multiple of 100, up to 900, with procedural reliability.</p> <p><b>Access Point</b> MA.3.NSO.2.AP.3 Explore multiplying a one-digit whole number by 10.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent multiplication situations using objects organized in equal groups or in rectangular arrays and use the representations to find the total.</li> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod).</li> <li>• Skip count by 10's up to 90.</li> </ul>
MA.3.NSO.2.4	<p>Multiply two whole numbers from 0 to 12 and divide using related facts with procedural reliability.</p> <p><b>Access Point</b> MA.3.NSO.2.AP.4 Explore the relationship between multiplication and division in order to multiply and divide. Multiplication may not exceed two single-digit whole numbers and their related division facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of equal groups.</li> <li>• Distinguish between the number of groups, the number in each group, and the total number.</li> <li>• Represent multiplication situations using objects organized in equal groups and use the representations to find the total.</li> <li>• Represent division situations by organizing</li> </ul>

objects in equal groups and use the representations to find the number of groups or the number in each group.

## Fractions

### **MA.3.FR.1 Understand fractions as numbers and represent fractions.**

MA.3.FR.1.1

Represent and interpret unit fractions in the form  $\frac{1}{n}$  as the quantity formed by one part when a whole is partitioned into  $n$  equal parts.

**Access Point**

MA.3.FR.1.AP.1 Explore unit fractions in the form  $\frac{1}{n}$  as the quantity formed by one part when a whole is partitioned into  $n$  equal parts. Denominators are limited to 2, 3 and 4.

**Essential Understandings:**

- Partition circles and rectangles into two, three or four equal-sized parts.
- Recognize the parts of the whole as halves, thirds, or fourths.

MA.3.FR.1.2

Represent and interpret fractions, including fractions greater than one, in the form of  $\frac{m}{n}$  as the result of adding the unit fraction  $\frac{1}{n}$  to itself  $m$  times.

**Access Point**

MA.3.FR.1.AP.2 Explore fractions, less than or equal to a whole, in the form of  $\frac{m}{n}$  as the result of adding the unit fraction  $\frac{1}{n}$  to itself  $m$  times. Denominators are limited to 2, 3 and 4.

**Essential Understandings:**

- Understand unit fractions in the form  $1/n$  as the quantity formed by one part when a whole is partitioned into  $n$  equal parts.

MA.3.FR.1.3

Read and write fractions, including fractions greater than one, using standard form, numeral-word form and word form.



	<p><b>Access Point</b></p> <p>MA.3.FR.1.AP.3 Read and generate fractions, less than or equal to a whole, using standard form.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize the parts of the whole as halves, thirds, or fourths.</li> <li>• Understand fractions, less than or equal to a whole, in the form of <math>m/n</math> is the result of adding the unit fraction <math>1/n</math> to itself <math>m</math> times.</li> </ul>
<p><b>MA.3.FR.2 Order and compare fractions and identify equivalent fractions.</b></p>	
MA.3.FR.2.1	<p>Plot, order and compare fractional numbers with the same numerator or the same denominator.</p> <p><b>Access Point</b></p> <p>MA.3.FR.2.AP.1 Compare fractional numbers with the same denominator. Denominators are limited to 2, 3 and 4.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described.</li> </ul>
MA.3.FR.2.2	<p>Identify equivalent fractions and explain why they are equivalent.</p> <p><b>Access Point</b></p> <p>MA.3.FR.2.AP.2 Using a visual model, recognize fractions less than a whole that are equivalent to fractions with denominators of 2, 3 or 4 (e.g., <math>\frac{4}{8}</math> is equivalent to <math>\frac{1}{2}</math>).</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that when a whole is partitioned into more parts, the parts are smaller and when a whole is partitioned into less parts, the parts are larger.</li> <li>• Understand that a greater quantity of smaller parts can be combined to cover the same area as a lesser quantity of larger parts.</li> </ul>

**Algebraic Reasoning**

<b>MA.3.AR.1 Solve multiplication and division problems.</b>	
MA.3.AR.1.1	Apply the distributive property to multiply a one-digit number and two-digit number. Apply properties of multiplication to find a product of one-digit whole numbers.
	<p><b>Access Point</b></p> <p>MA.3.AR.1.AP.1 Apply the commutative property of multiplication to find a product of one-digit whole numbers.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent multiplication expressions using objects to find products.</li> <li>• Recognize that when given a multiplication expression that changing the order of the factors does not change the product.</li> </ul>
MA.3.AR.1.2	Solve one- and two-step real-world problems involving any of four operations with whole numbers.
	<p><b>Access Point</b></p> <p>MA.3.AR.1.AP.2a Solve one- and two-step addition and subtraction real-world problems within 100.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” with objects or drawings.</li> <li>• Understand the need to represent all actions in a situation and that there may be more than one action required.</li> <li>• Add or subtract within 100.</li> </ul>
	MA.3.AR.1.AP.2b Solve one-step multiplication and division real-world problems. Multiplication may not exceed two single-digit whole numbers and their related division facts.
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent multiplication and division situations involving equal groups and rectangular arrays with objects or drawings.</li> <li>• Multiply two single-digit whole numbers and perform their related division facts.</li> </ul>

**MA.3.AR.2 Develop an understanding of equality and multiplication and division.**

MA.3.AR.2.1	<p>Restate a division problem as a missing factor problem using the relationship between multiplication and division.</p> <p><b>Access Point</b></p> <p>MA.3.AR.2.AP.1 Explore division as multiplication with a missing factor using the relationship between multiplication and division.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"><li>• Model multiplication and division expressions with objects.</li><li>• Given a multiplication or division expression (e.g., <math>4 \times 3</math>; <math>12 \div 4</math>), use objects to perform multiplication of two single-digit whole numbers and their related division facts.</li></ul>
MA.3.AR.2.2	<p>Determine and explain whether an equation involving multiplication or division is true or false.</p> <p><b>Access Point</b></p> <p>MA.3.AR.2.AP.2 Determine if multiplication or division equations with no more than three terms are true or false. Multiplication may not exceed two single-digit whole numbers and their related division facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"><li>• Use objects to find products of two single-digit whole numbers and their related division facts.</li><li>• Understand the concept of "equality" as the balance of two values (e.g., if a balance scale is level, then the values are equal and if it is not level, then the values are not equal).</li><li>• Understand that = is "equal to".</li><li>• Understand that if the values on either side of the equal sign are the same, then the equation is true and if the values on either side of the equal side are not the same, then the equation is false.</li></ul>
MA.3.AR.2.3	<p>Determine the unknown whole number in a multiplication or division equation, relating three whole numbers, with the unknown in any position.</p> <p><b>Access Point</b></p>

MA.3.AR.2.AP.3 Determine the unknown whole number in a multiplication or division equation, relating three whole numbers, with the product or quotient unknown (e.g.,  $2 \times 5 = \underline{\quad}$ ,  $10 \div 5 = \underline{\quad}$ ). Multiplication may not exceed two single-digit whole numbers and their related division facts.

**Essential Understandings:**

- Given a multiplication or division expression (e.g.,  $2 \times 5$ ;  $10 \div 5$ ) use objects to solve.
- Understand a symbol (e.g.,  $\underline{\quad}$  or  $\square$ ) may be used to represent an unknown number in an equation.
- Understand that  $=$  is "equal to".

***MA.3.AR.3 Identify numerical patterns, including multiplicative patterns.***

MA.3.AR.3.1

Determine and explain whether a whole number from 1 to 1,000 is even or odd.

**Access Point**

MA.3.AR.3.AP.1 Determine whether a whole number from 1 to 100 is even or odd.

**Essential Understandings:**

- Recognize that even numbers can be paired using two equal groups and odd numbers can be paired with two equal groups with one left over.
- Recognize that there is a pattern where you only have to look in the ones place to determine if a number is odd or even.

MA.3.AR.3.2

Determine whether a whole number from 1 to 144 is a multiple of a given one-digit number.

**Access Point**

MA.3.AR.3.AP.2 Explore that a whole number is a multiple of each of its factors. Factors not to exceed single-digit whole numbers.

**Essential Understandings:**

- Understand the concept of multiplication involves the accumulation of equal groups.

MA.3.AR.3.3

Identify, create and extend numerical patterns.

**Access Point**

MA.3.AR.3.AP.3 Extend a numerical pattern when given a one-step addition rule (e.g., when given the pattern 5, 10, 15, use the rule add 5 to extend the pattern).

**Essential Understandings:**

- Understand that patterns are repeated and predictable.
- Perform basic addition.

## Measurement

***MA.3.M.1 Measure attributes of objects and solve problems involving measurement.***

MA.3.M.1.1

Select and use appropriate tools to measure the length of an object, the volume of liquid within a beaker and temperature.

**Access Point**

MA.3.M.1.AP.1a Select and use appropriate tools to measure the length (i.e., inches, feet, yards) of an object.

**Essential Understandings:**

- Understand that length is an attribute of objects that can be measured using a ruler or yard stick and the length of the object being measured influences the choice of the tool (i.e., use a ruler to measure the length of a pencil and use a yard stick to measure the length of the classroom).
- Understand that length is an attribute that can be measured in inches, feet, and yards.
- Identify the beginning and end point of the object that needs to be measured.
- Recognize that the units marked on a ruler/yard stick have equal length intervals.
- Understand that the total number of equal interval distances, spanned end to end, can be counted to determine the overall length of an object.

MA.3.M.1.AP.1b Explore selecting and using appropriate tools to measure liquid volume (i.e., gallons, quarts, pints, cups) and temperature in degrees Fahrenheit.

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that measurement tools are selected based on the attribute being measured</li> </ul>
MA.3.M.1.2	<p>Solve real-world problems involving any of the four operations with whole-number lengths, masses, weights, temperatures or liquid volumes.</p> <p><b>Access Point</b></p> <p>MA.3.M.1.AP.2a Solve one- and two-step addition and subtraction real-world problems within 100 with whole number lengths (i.e., inches, feet, yards), temperatures (i.e., degrees Fahrenheit) or liquid volumes (i.e., gallons, quarts, pints, cups).</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction measurement situations involving “adding to” and “taking from” with objects or drawings.</li> <li>• Understand the need to represent all actions in a situation and that there may be more than one action required.</li> <li>• Add or subtract within 10</li> </ul> <p>MA.3.M.1.AP.2b Solve one-step multiplication and division real-world problems with whole number lengths (i.e., inches, feet, yards), temperatures (i.e., degrees Fahrenheit) or liquid volumes (i.e., gallons, quarts, pints and cups). Multiplication may not exceed two single-digit whole numbers and their related division facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent multiplication and division measurement situations with objects or drawings.</li> <li>• Multiply two single-digit whole numbers and perform their related division.</li> </ul>
<p><b><i>MA.3.M.2 Tell and write time and solve problems involving time.</i></b></p>	
MA.3.M.2.1	<p>Using analog and digital clocks, tell and write time to the nearest minute using a.m. and p.m. appropriately.</p> <p><b>Access Point</b></p> <p>MA.3.M.2.AP.1 Using analog and digital clocks, express</p>

the time to the nearest five minutes using a.m. and p.m. appropriately.

**Essential Understandings:**

- Understand that time is an attribute that can be measured with a clock and can be expressed in hours and minutes.
- Recognize that on an analog clock the longer hand is the minute hand and that the shorter hand is the hour hand.
- Understand that when the shorter hand starts at 12 and moves one full rotation around the clock back to 12 that 60 minutes (or an hour) has passed and that a new hour begins.
- Skip count by 5's (up to 55).
- Recognize that on an analog clock the last numeral that the shorter hand reached/passed represents the hours (The time is read as hour then minutes).
- Recognize that on a digital clock the numerals 1-12, before the colon, represent the hours and the numerals (00-59) after the colon represent the minutes (The time is read as hour then minutes).
- Understand that when telling time, it is important to specify whether the time is a.m. or p.m.

MA.3.M.2.2

Solve one- and two-step real-world problems involving elapsed time.

**Access Point**

MA.3.M.2.AP.2 Solve for end time in one-step real-world problems when given start time and elapsed time in whole hours or minutes within the hour.

**Essential Understandings:**

- Represent situations involving "adding to" with objects or drawings.
- Add within 60.

**Geometric Reasoning**

***MA.3.GR.1 Describe and identify relationships between lines and***

<b><i>classify quadrilaterals.</i></b>	
MA.3.GR.1.1	Describe and draw points, lines, line segments, rays, intersecting lines, perpendicular lines and parallel lines. Identify these in two-dimensional figures.
	<b>Access Point</b> MA.3.GR.1.AP.1 Identify points, lines, line segments, perpendicular lines and parallel lines. Identify these in two-dimensional figures.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand the terms “points,” “lines,” “line segments,” “perpendicular lines,” and “parallel lines”.</li> </ul>
MA.3.GR.1.2	Identify and draw quadrilaterals based on their defining attributes. Quadrilaterals include parallelograms, rhombi, rectangles, squares and trapezoids.
	<b>Access Point</b> MA.3.GR.1.AP.2 Identify quadrilaterals based on their defining attributes. Quadrilaterals include parallelograms, rhombi, rectangles, squares, and trapezoids.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Identify specified defining attributes (i.e., sides, vertices, closed versus open, straight versus curved) in isolated quadrilaterals.</li> <li>• Understand the defining attributes of quadrilaterals.</li> </ul>
MA.3.GR.1.3	Draw line(s) of symmetry in a two-dimensional figure and identify line-symmetric two-dimensional figures.
	<b>Access Point</b> MA.3.GR.1.AP.3 Identify line-symmetric two-dimensional figures.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Recognize when a shape can be divided into two equal parts.</li> <li>• Understand the concept of a line of symmetry.</li> </ul>
<b><i>MA.3.GR.2 Solve problems involving the perimeter and area of rectangles.</i></b>	
MA.3.GR.2.1	Explore area as an attribute of a two-dimensional figure



	<p>by covering the figure with unit squares without gaps or overlaps. Find areas of rectangles by counting unit squares.</p> <p><b>Access Point</b></p> <p>MA.3.GR.2.AP.1 Explore area as an attribute of a two-dimensional figure that can be measured by covering the figure with unit squares without gaps or overlaps.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Express the length of a side of an object as a whole number of lengths using non-standard objects laid end to end with no gaps or overlaps.</li> </ul>
MA.3.GR.2.2	<p>Find the area of a rectangle with whole-number side lengths using a visual model and a multiplication formula.</p> <p><b>Access Point</b></p> <p>MA.3.GR.2.AP.2 Find the area of a rectangle with whole-number side lengths by counting unit squares. Explore that the area is the same as what would be found by multiplying the side lengths.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand the concept of area</li> <li>Understand the concept of multiplication using arrays</li> </ul>
MA.3.GR.2.3	<p>Solve mathematical and real-world problems involving the perimeter and area of rectangles with whole-number side lengths using a visual model and a formula.</p> <p><b>Access Point</b></p> <p>MA.3.GR.2.AP.3 Solve mathematical and real-world problems involving the perimeter and area of rectangles with whole-number side lengths using a visual model.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Distinguish between the concepts of area and perimeter.</li> <li>Find the perimeter of a rectangle with whole-number side lengths given.</li> <li>Find the area of a rectangle with whole-number side lengths by counting unit squares or multiplying the side lengths.</li> </ul>

MA.3.GR.2.4	Solve mathematical and real-world problems involving the perimeter and area of composite figures composed of non-overlapping rectangles with whole-number side lengths.
	<p><b>Access Point</b></p> <p>MA.3.GR.2.AP.4 Explore the perimeter and area of composite figures composed of two non-overlapping rectangles with whole-number side lengths.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Distinguish between the concepts of area and perimeter.</li> <li>• Find the perimeter of a rectangle with whole-number side lengths given.</li> <li>• Find the area of a rectangle with whole-number side lengths by counting unit squares or multiplying the side lengths</li> </ul>

### **Data Analysis and Probability**

<b><i>MA.3.DP.1 Collect, represent and interpret numerical and categorical data.</i></b>	
MA.3.DP.1.1	Collect and represent numerical and categorical data with whole-number values using tables, scaled pictographs, scaled bar graphs or line plots. Use appropriate titles, labels and units.
	<p><b>Access Point</b></p> <p>MA.3.DP.1.AP.1a Sort and represent categorical data (up to four categories) with whole-number values using tables, pictographs or bar graphs. Select appropriate title, labels and units.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that each category represents a group with a characteristic in common</li> <li>• Understand that each tally mark or picture represents one data point from that category</li> <li>• Understand that the total number of tally marks or pictures in each category tells “how many” in each category.</li> <li>• Understand that the numerals in each section of the table or the height of each bar tells “how many” in each category</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand the terms and location of “title” “labels” and “units”</li> </ul> <p>MA.3.DP.1.AP.1b Explore representing numerical data with whole-number values using line plots.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand using a horizontal number line</li> <li>• Understand that different types of data can be collected and represented in various ways</li> </ul>
MA.3.DP.1.2	<p>Interpret data with whole-number values represented with tables, scaled pictographs, circle graphs, scaled bar graphs or line plots by solving one- and two-step problems.</p> <p><b>Access Point</b></p> <p>MA.3.DP.1.AP.2a Interpret data with whole-number values represented with tables, pictographs or bar graphs to solve one-step “how many more” and “how many less” problems.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that each category represents a group with a characteristic in common</li> <li>• Understand that each tally mark or picture represents one data point from that category</li> <li>• Understand that the total number of tally marks or pictures in each category tells “how many” in each category</li> <li>• Understand that the numerals in each section of the table or the height of each bar tells “how many” in each category</li> <li>• Understand the terms and location of “title” “labels” and “units”</li> <li>• Understand the concepts of “more” and “less”</li> <li>• Use objects or drawings to solve comparison problems</li> </ul> <p>MA.3.DP.1.AP.2b Interpret data with whole-number values represented with scaled pictographs or scaled bar graphs. For scaled pictographs, symbols used may only represent quantities of 2, 5 or 10 and only whole symbols may be used. For scaled bar graphs, intervals may only represent quantities of 2, 5 or 10.</p>

**Essential Understandings:**

- Understand that each category represents a group with a characteristic in common
- Understand that each picture represents data from that category
- Understand that the total value of the pictures in each category tells “how many” in each category.
- Skip count by 2’s, 5’s, and 10’s
- Understand that the height of each bar tells “how many” in each category
- Understand the terms and location of “title” “labels,” “units,” and “key”

MA.3.DP.1.AP.2c Explore interpreting data with whole-number values represented with line plots.

**Essential Understandings:**

- Understand reading a horizontal number line
- Understand that each X or dot on the line plot represents 1 object with that length, temperature, or liquid volume
- Use repeated addition of whole numbers to find totals

**Grade 4 B.E.S.T. Standards Access Points  
Number Sense and Operations**

***MA.4.NSO.1 Understand place value for multi-digit numbers.***

MA.4.NSO.1.1 Express how the value of a digit in a multi-digit whole number changes if the digit moves one place to the left or right.

**Access Point**

MA.4.NSO.1.AP.1 Explore how the value of a digit in a multi-digit whole number changes if the digit moves one place to the left.

**Essential Understandings:**

- Understand that 10 ones is equal to 1 ten, 10 tens is equal to 1 hundred, 10 hundreds is equal to 1 thousand, and 10 thousands is equal to 1 ten-thousands
- Recognize the location of the ten-thousands

	<p>place, the thousands place, the hundreds place, the tens place, and the ones place</p> <ul style="list-style-type: none"> <li>• Understand that the digit in the ten-thousands place represents the number of ten-thousands, the digit in the thousands place represents the number of thousands, the digit in the hundreds place represents the number of hundreds, the digit in the tens place represents the number of tens, and the digit in the ones place represents the number of ones</li> </ul>
<p>MA.4.NSO.1.2</p>	<p>Read and write multi-digit whole numbers from 0 to 1,000,000 using standard form, expanded form and word form.</p> <p><b>Access Point</b></p> <p>MA.4.NSO.1.AP.2 Read and generate numbers from 0 to 10,000 using standard form and expanded form.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Express number names (rote count) up to 100</li> <li>• Skip count by 100's up to 1,000</li> <li>• Skip count by 1,000's up 10,000</li> <li>• Identify a number written in standard form when given the name of the number up to 1,000</li> <li>• Understand that the 5 digits of a five-digit number represent an amount of ten-thousands, thousands, hundreds, tens, and further ones</li> <li>• Understand that expanded form is the value of the ten-thousands, plus the value of the thousands, plus the value of the hundreds, plus the value of the tens, plus the value of the ones.</li> <li>• Understand that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to 1, 2, 3, 4, 5, 6, 7, 8, 9 tens and 0 further ones</li> <li>• Understand that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to 1, 2, 3, 4, 5, 6, 7, 8, 9 hundreds and 0 further tens and 0 further ones</li> <li>• Understand that the numbers 1,000; 2,000; 3,000; 4,000; 5,000; 6,000; 7,000; 8,000;</li> </ul>

	<p>9,000 refer to 1, 2, 3, 4, 5, 6, 7, 8, 9 thousands, and 0 further hundreds and 0 further tens and 0 further ones</p> <ul style="list-style-type: none"> <li>• Generate numbers 0-1,000 using standard form</li> </ul>
<p>MA.4.NSO.1.3</p>	<p>Plot, order and compare multi-digit whole numbers up to 1,000,000.</p> <p><b>Access Point</b></p> <p>MA.4.NSO.1.AP.3 Plot, order and compare multi-digit whole numbers up to 10,000.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use visuals to represent numbers up to 10,000 using ten thousands, thousands, hundreds, tens, and ones</li> <li>• Use matching of same unit visuals to compare starting with the ten thousands place</li> <li>• Understand that <math>&gt;</math> is "greater than", <math>&lt;</math> is "less than", and <math>=</math> is "equal to"</li> <li>• Understand that numbers on a number line are plotted in sequential order, numbers that are lower on the number line have a lesser value, and numbers higher on the number line have a greater value</li> </ul>
<p>MA.4.NSO.1.4</p>	<p>Round whole numbers from 0 to 10,000 to the nearest 10, 100 or 1,000.</p> <p><b>Access Point</b></p> <p>MA.4.NSO.1.AP.4 Round whole numbers from 100 to 10,000 to the nearest 1,000 with visual support.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a number up to 10,000, understand that the digit in the ten thousands place represents the number of ten thousands, the digit in the thousands place represents the number of thousands, the digit in the hundreds place represents the number of hundreds, the digit in the tens place represents the number of tens, and the digit in the ones place represents the number of ones</li> <li>• Understand that numbers on a number line are plotted in sequential order, numbers that</li> </ul>

	<p>are lower on the number line have a lesser value, and numbers higher on the number line have a greater value</p> <ul style="list-style-type: none"> <li>• Plot whole numbers up to 10,000 on a number line</li> <li>• Identify which millennium the number being rounded is closest to on a number line</li> <li>• Understand that if the number being rounded is halfway between two millenniums, then it rounds to the greater millennium</li> </ul>
MA.4.NSO.1.5	Plot, order and compare decimals up to the hundredths.
	<b>Access Point</b> MA.4.NSO.1.AP.5 Using visual models, compare decimals less than one up to the hundredths.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Recognize that decimals are parts of a whole</li> </ul>
<b><i>MA.4.NSO.2 Build an understanding of operations with multi-digit numbers including decimals.</i></b>	
MA.4.NSO.2.1	Recall multiplication facts with factors up to 12 and related division facts with automaticity.
	<b>Access Point</b> MA.4.NSO.2.AP.1 Recall multiplication facts of one-digit whole numbers multiplied by 1, 2, 5 and 10.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Represent multiplication expressions (e.g., <math>2 \times 5</math>) using objects or drawings organized in equal groups or in rectangular arrays and use the representations to find the total</li> </ul>
MA.4.NSO.2.2	Multiply two whole numbers, up to three digits by up to two digits, with procedural reliability.
	<b>Access Point</b> MA.4.NSO.2.AP.2 Explore multiplication of two whole numbers, up to two digits by one digit.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod)</li> <li>• Model two-digit numbers using 10s and 1s</li> </ul>

	<p>(e.g., ten-rods and unit cubes)</p> <ul style="list-style-type: none"> <li>• Represent single-digit-by-single-digit multiplication situations using objects organized in equal groups or in rectangular arrays and use the representations to find the total</li> </ul>
MA.4.NSO.2.3	Multiply two whole numbers, each up to two digits, including using a standard algorithm with procedural fluency.
	<p><b>Access Point</b></p> <p>MA.4.NSO.2.AP.3 Apply a strategy to multiply two whole numbers up to two digits by one digit.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent multiplication expressions (e.g., <math>3 \times 12</math>) using objects or drawings organized in equal groups or in rectangular arrays and use the representations to find the total</li> </ul>
MA.4.NSO.2.4	Divide a whole number up to four digits by a one-digit whole number with procedural reliability. Represent remainders as fractional parts of the divisor.
	<p><b>Access Point</b></p> <p>MA.4.NSO.2.AP.4 Explore division of two whole numbers up to two digits by one digit with and without remainders. Represent remainders as whole numbers.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod)</li> <li>• Model two-digit numbers using 10s and 1s (e.g., ten-rods and unit cubes)</li> <li>• Represent division situations related to single-digit multiplication using objects organized in equal groups and use the representations to find the total number of groups or the number in each group</li> </ul>
MA.4.NSO.2.5	Explore the multiplication and division of multi-digit whole numbers using estimation, rounding and place value.
	<p><b>Access Point</b></p>



MA.4.NSO.2.AP.5 Explore the estimation of products and quotients of two whole numbers up to two digits by one digit.

**Essential Understandings:**

- Round two-digit numbers in an expression to the nearest 10 to create a simpler problem
- Represent multiplication expressions (e.g.,  $3 \times 20$ ) using objects or drawings organized in equal groups and use the representations to find the total.
- Represent division expressions (e.g.,  $60 \div 3$ ) using objects or drawings organized in equal groups and use the representations to find the number of groups or the number in each group
- Recognize that rounding two-digit numbers in an expression prior to multiplying or dividing provides an estimation of a reasonable solution without performing the exact computations required to solve the problem

MA.4.NSO.2.6

Identify the number that is one-tenth more, one-tenth less, one-hundredth more and one-hundredth less than a given number.

**Access Point**

MA.4.NSO.2.AP.6 Identify the number that is one-tenth more and one-tenth less than a given number (i.e., 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9).

**Essential Understandings:**

- Understand that the digit in the ones place represents the number of ones and the digit in the tenths place represents the number of tenths
- Use objects (e.g., tenth rods) to represent the numbers 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
- Understand that "more" increases the number of tenths and that "less" decreases the number of tenths
- Understand that a group of 10 tenths is equal to 1 whole (e.g., 10 tenth rods is equal to 1)

	whole flat)
MA.4.NSO.2.7	Explore the addition and subtraction of multi-digit numbers with decimals to the hundredths.
	<b>Access Point</b> MA.4.NSO.2.AP.7 Explore the addition and subtraction of decimals less than one to the tenths (e.g., $0.3 + 0.5$ ) and hundredths (e.g., $0.25 - 0.12$ ).
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand that the digit in the ones place represents the number of ones, the digit in the tenths place represents the number of tenths, and the digit in the hundredths place represents the number of hundredths</li> <li>• Use objects (e.g., tenth rods and hundredth unit cubes) to represent decimals less than one to the tenths and hundredths</li> <li>• Understand that a group of 10 tenths is equal to 1 whole (e.g., 10 tenth rods is equal to 1 whole flat) and that 1 whole is equal to a group of 10 tenths (e.g., 1 whole flat is equal to 10 tenth rods)</li> <li>• Understand that a group of 10 hundredths is equal to 1 tenth (e.g., 10 hundredth unit cubes is equal to 1 tenth rod) and that 1 tenth is equal to 10 hundredths (e.g., 1 tenth rod is equal to 10 hundredth unit cubes)</li> <li>• Understand that when adding or subtracting like place value units are added or subtracted</li> </ul>

## Fractions

<b><i>MA.4.FR.1 Develop an understanding of the relationship between different fractions and the relationship between fractions and decimals.</i></b>	
MA.4.FR.1.1	Model and express a fraction, including mixed numbers and fractions greater than one, with the denominator 10 as an equivalent fraction with the denominator 100.
	<b>Access Point</b> MA.4.FR.1.AP.1 Using a visual model, recognize fractions less than one, with the denominator 10 as

	<p>an equivalent fraction with the denominator 100 (e.g., <math>\frac{2}{10}</math> is equivalent to <math>\frac{20}{100}</math>).</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that when a whole is partitioned into more parts, the parts are smaller and when a whole is partitioned into less parts, the parts are larger</li> <li>• Understand that a greater quantity of smaller parts can be combined to cover the same area as a lesser quantity of larger parts</li> </ul>
MA.4.FR.1.2	<p>Use decimal notation to represent fractions with denominators of 10 or 100, including mixed numbers and fractions greater than 1, and use fractional notation with denominators of 10 or 100 to represent decimals.</p> <p><b>Access Point</b></p> <p>MA.4.FR.1.AP.2 Use decimal notation to represent fractions less than one with denominators of 10 or 100 and use fractional notation with denominators of 10 or 100 to represent decimals less than one.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that fractions and decimals can be used to describe parts of a whole</li> <li>• Understand that a tenth is one-tenth (<math>\frac{1}{10}</math> or 0.1) of a whole</li> <li>• Understand that a hundredth is one-hundredth (<math>\frac{1}{100}</math> or 0.01) of a whole</li> <li>• Use objects to represent numbers less than one using tenths and hundredths</li> <li>• Given a fraction less than 1, understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described</li> <li>• Given a decimal less than 1, understand that the digit in the ones place represents the number of ones, the digit in the tenths place represents the number of tenths, the digit in the hundredths place represents then number of hundredths</li> </ul>
MA.4.FR.1.3	Identify and generate equivalent fractions, including

	<p>fractions greater than one. Describe how the numerator and denominator are affected when the equivalent fraction is created.</p>
	<p><b>Access Point</b></p> <p>MA.4.FR.1.AP.3 Using a visual model, generate fractions less than a whole that are equivalent to fractions with denominators 2, 3, 4, 6, 8 or 10. Explore how the numerator and denominator are affected when the equivalent fraction is created.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that when a whole is partitioned into more parts, the parts are smaller and when a whole is partitioned into less parts, the parts are larger</li> <li>• Understand that a greater quantity of smaller parts can be combined to cover the same area as a lesser quantity of larger parts</li> <li>• Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described</li> </ul>
<p>MA.4.FR.1.4</p>	<p>Plot, order and compare fractions, including mixed numbers and fractions greater than one, with different numerators and different denominators.</p>
	<p><b>Access Point</b></p> <p>MA.4.FR.1.AP.4a Explore mixed numbers and fractions greater than one.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand fractions in the form of <math>m/n</math> is the result of adding the unit fraction <math>1/n</math> to itself <math>m</math> times</li> <li>• Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described</li> <li>• Understand that if the number of equal parts being described is the same as the number of equal parts in the whole, then the fraction is equal to 1</li> </ul>
	<p>MA.4.FR.1.AP.4b Using visual models, compare fractions less than one with different numerators and different denominators. Denominators limited to 2, 3, 4, 6, 8 or 10.</p>

**Essential Understandings:**

- Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described
- Understand that when a whole is partitioned into more parts, the parts are smaller and when a whole is partitioned into less parts, the parts are larger
- Understand that a greater quantity of smaller parts can be combined to cover the same area as a lesser quantity of larger parts
- Compare fractional numbers with the same denominator

**MA.4.FR.2 Build a foundation of addition, subtraction and multiplication operations with fractions.**

MA.4.FR.2.1

Decompose a fraction, including mixed numbers and fractions greater than one, into a sum of fractions with the same denominator in multiple ways. Demonstrate each decomposition with objects, drawings and equations.

**Access Point**

MA.4.FR.2.AP.1 Decompose a fraction less than one into a sum of unit fractions with the same denominator (e.g.,  $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ ). Denominators limited to 2, 3, 4, 6, 8 or 10. Demonstrate each decomposition with objects, drawings or equations.

**Essential Understandings:**

- Understand fractions in the form of  $m/n$  is the result of adding the unit fraction  $1/n$  to itself  $m$  times

MA.4.FR.2.2

Add and subtract fractions with like denominators, including mixed numbers and fractions greater than one, with procedural reliability.

**Access Point**

MA.4.FR.2.AP.2 Explore adding and subtracting fractions less than one with like denominators. Denominators limited to 2, 3, 4, 6, 8 or 10.

**Essential Understandings:**

- Understand the denominator is the size of the equal parts of the whole and the numerator is

	<p>the number of equal parts being described</p> <ul style="list-style-type: none"> <li>• Understand fractions in the form of <math>m/n</math> is the result of adding the unit fraction <math>1/n</math> to itself <math>m</math> times</li> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” with objects</li> </ul>
<p>MA.4.FR.2.3</p>	<p>Explore the addition of a fraction with denominator of 10 to a fraction with denominator of 100 using equivalent fractions.</p> <p><b>Access Point</b></p> <p>MA.4.FR.2.AP.3 Explore the addition of a fraction with denominator of 10 to a fraction with denominator of 100 using visual models to find equivalent fractions.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described</li> <li>• Understand fractions in the form of <math>m/n</math> is the result of adding the unit fraction <math>1/n</math> to itself <math>m</math> times</li> <li>• Understand that when a whole is partitioned into more parts, the parts are smaller and when a whole is partitioned into less parts, the parts are larger</li> <li>• Understand that a greater quantity of smaller parts can be combined to cover the same area as a lesser quantity of larger parts</li> <li>• Recognize fractions less than one, with the denominator 10 as an equivalent fraction with the denominator 100 (e.g., <math>2/10</math> is equivalent to <math>20/100</math>)</li> <li>• Represent addition situations involving “adding to” with objects</li> </ul>
<p>MA.4.FR.2.4</p>	<p>Extend previous understanding of multiplication to explore the multiplication of a fraction by a whole number or a whole number by a fraction.</p> <p><b>Access Point</b></p> <p>MA.4.FR.2.AP.4 Explore the multiplication of a unit fraction</p>

by a whole number (e.g.,  $3 \times \frac{1}{4}$ ,  $2 \times \frac{1}{6}$ ,  $5 \times \frac{1}{2}$ ).  
 Denominators limited to 2, 3, 4, 6, 8 or 10.

**Essential Understandings:**

- Understand the concept of equal groups
- Distinguish between the number of groups and the number in each group
- Recognize in multiplication situations that the number in each group is the same and can be repeatedly added to find the total
- Represent multiplication situations using objects organized in equal groups and use the representations to find the total
- Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described
- Understand fractions in the form of  $m/n$  is the result of adding the unit fraction  $1/n$  to itself  $m$  times

**Algebraic Reasoning**

***MA.4.AR.1 Represent and solve problems involving the four operations with whole numbers and fractions.***

MA.4.AR.1.1

Solve real-world problems involving multiplication and division of whole numbers including problems in which remainders must be interpreted within the context.

**Access Point**

MA.4.AR.1.AP.1 Solve one-step real-world problems involving multiplication and division of whole numbers. Multiplication may not exceed two-digit by one-digit and division must be related to one-digit by one-digit multiplication facts.

**Essential Understandings:**

- Represent multiplication and division situations involving equal groups and rectangular arrays with objects or drawings
- Multiply two-digit by one-digit whole numbers
- Perform division related to one-digit by one-digit multiplication facts

<p>MA.4.AR.1.2</p>	<p>Solve real-world problems involving addition and subtraction of fractions with like denominators, including mixed numbers and fractions greater than one.</p> <p><b>Access Point</b></p> <p>MA.4.AR.1.AP.2 Solve one-step real-world problems involving addition and subtraction of fractions less than one with like denominators. Denominators limited to 2, 3, 4, 6, 8 or 10.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations with drawings or objects</li> <li>• Add or subtract fractions less than one with like denominators limited to 2, 3, 4, 6, 8, or 10</li> </ul>
<p>MA.4.AR.1.3</p>	<p>Solve real-world problems involving multiplication of a fraction by a whole number or a whole number by a fraction.</p> <p><b>Access Point</b></p> <p>MA.4.AR.1.AP.3 Solve one-step real-world problems involving multiplication of a unit fraction by a whole number (e.g., <math>3 \times \frac{1}{4}</math>, <math>2 \times \frac{1}{6}</math>, <math>5 \times \frac{1}{2}</math>). Denominators limited to 2, 3, 4, 6, 8 or 10.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent situations involving multiplication with drawings or objects</li> <li>• Multiply a unit fraction by a whole number with denominators limited to 2, 3, 4, 6, 8, or 10</li> </ul>
<p><b><i>MA.4.AR.2 Demonstrate an understanding of equality and operations with whole numbers.</i></b></p>	
<p>MA.4.AR.2.1</p>	<p>Determine and explain whether an equation involving any of the four operations with whole numbers is true or false.</p> <p><b>Access Point</b></p> <p>MA.4.AR.2.AP.1 Determine whether an equation (with no more than three terms) involving any of the four operations with whole numbers is true or false. Sums may not exceed 100 and their related subtraction facts. Multiplication may not exceed two-digit by one-digit and division must be related to one-digit by one-digit</p>



	<p>multiplication facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Find sums within 100 and their related subtraction facts</li> <li>• Find products of two-digit by one-digit whole numbers</li> <li>• Find quotients of related one-digit by one-digit multiplication facts.</li> <li>• Understand the concept of “equality” as the balance of two values (e.g., if a balance scale is level, then the values are equal and if it is not level, then the values are not equal)</li> <li>• Understand that = is “equal to”</li> <li>• Understand that if the values on either side of the equal sign are the same, then the equation is true and if the values on either side of the equal side are not the same, then the equation is false</li> </ul>
<p>MA.4.AR.2.2</p>	<p>Given a mathematical or real-world context, write an equation involving multiplication or division to determine the unknown whole number with the unknown in any position.</p> <p><b>Access Point</b></p> <p>MA.4.AR.2.AP.2 Given a real-world context, identify or generate an equation involving multiplication or division to determine the unknown product or quotient. Multiplication may not exceed two-digit by one-digit and division must be related to one-digit by one-digit multiplication facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand <math>\times</math> as a symbol representing the operation of multiplication and <math>\div</math> as a symbol representing the operation of division</li> <li>• Understand = as a symbol representing the equality of two values</li> <li>• Understand a symbol (e.g., ___ or <math>\square</math>) may be used to represent an unknown number in an equation</li> <li>• Interpret relevant information in a real-world context</li> <li>• Find products of two-digit by one-digit whole numbers</li> </ul>

	<ul style="list-style-type: none"> <li>Find quotients of related one-digit by one-digit multiplication facts</li> </ul>
<b>MA.4.AR.3 Recognize numerical patterns, including patterns that follow a given rule.</b>	
MA.4.AR.3.1	Determine factor pairs for a whole number from 0 to 144. Determine whether a whole number from 0 to 144 is prime, composite or neither.
	<b>Access Point</b>
	MA.4.AR.3.AP.1 Explore factor pairs for a whole number. Factors may not exceed single-digit whole numbers.
	<b>Essential Understandings:</b>
	<ul style="list-style-type: none"> <li>Understand products can be represented as the accumulation of equal groups and may be represented in more than one way</li> </ul>
MA.4.AR.3.2	Generate, describe, and extend a numerical pattern that follows a given rule.
	<b>Access Point</b>
	MA.4.AR.3.AP.2 Generate a numerical pattern when given a starting term and a one-step addition rule (e.g., starting at the number 5 use the rule add 5 and generate the pattern).
	<b>Essential Understandings:</b>
	<ul style="list-style-type: none"> <li>Understand that patterns are repeated and predictable</li> <li>Perform basic addition</li> </ul>

### Measurement

<b>MA.4.M.1 Measure the length of objects and solve problems involving measurement.</b>	
MA.4.M.1.1	Select and use appropriate tools to measure attributes of objects.
	<b>Access Point</b>
	MA.4.M.1.AP.1a Select and use appropriate tools to measure length (i.e., inches, feet, yards), liquid volume (i.e., gallons, quarts, pints, cups) and temperature (i.e., degrees Fahrenheit).

**Essential Understandings:**

- Understand that length is an attribute of objects that can be measured using a ruler or yard stick and the length of the object being measured influences the choice of the tool (i.e., use a ruler to measure the length of a pencil and use a yard stick to measure the length of the classroom)
- Understand that length is an attribute that can be measured in inches, feet, and yards
- Understand that liquid volume is an attribute that can be measured using measuring spoons/cups and the amount of liquid being measured influences the choice of the size of the tool
- Understand that liquid volume is an attribute that can be measured in gallons, quarts, pints, and cups
- Understand that temperature is an attribute that can be measured using a thermometer
- Understand that temperature is an attribute that can be measured in degrees Fahrenheit
- Recognize the end point of a measurement reflects the total measure

MA.4.M.1.AP.1b Explore selecting and using appropriate tools to measure weight (i.e., ounces, pounds).

**Essential Understandings:**

- Understand that weight is an attribute of objects that can be measured, and the weight of the object being measured influences the choice of the measurement tool

MA.4.M.1.2

Convert within a single system of measurement using the units: yards, feet, inches; kilometers, meters, centimeters, millimeters; pounds, ounces; kilograms, grams; gallons, quarts, pints, cups; liter, milliliter; and hours, minutes, seconds.

**Access Point**

MA.4.M.1.AP.2a Explore relative sizes of measurement units within one system of units including yards, feet, inches; pounds, ounces; gallons, quarts, pints, cups; and

hours, minutes.

**Essential Understandings:**

- Understand that length is an attribute that can be measured in yards, feet, and inches that the amount of length of an object influences the unit selected for measurement
- Understand that weight is an attribute that can be measured in ounces and pounds and that the amount of weight of an object influences the unit selected for measurement
- Understand that liquid volume is an attribute that can be measured in gallons, quarts, pints, and cups and that the amount of liquid volume influences the unit selected for measurement
- Understand that time is an attribute that can be measured in hours and minutes and that the amount of time influences the unit selected for measurement

MA.4.M.1.AP.2b Using a conversion sheet, convert from a larger to a smaller unit within a single system of measurement using the units: yards, feet, inches; pounds, ounces; gallons, quarts, pints, cups; and hours, minutes. Only whole number measurements may be used.

**Essential Understandings:**

- Understand the relationship between the size of units of measurements within the same system of units (e.g., yards are longer than inches; pounds are heavier than ounces; gallons hold more than a pint; hours are longer than minutes)

***MA.4.M.2 Solve problems involving time and money.***

MA.4.M.2.1

Solve two-step real-world problems involving distances and intervals of time using any combination of the four operations.

**Access Point**

MA.4.M.2.AP.1a Solve one- and two-step real-world problems involving distances (i.e., inches, feet, yards, miles) in whole numbers using any combination of the four operations.

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent situations using any of the four operations with objects or drawings</li> <li>• Understand the need to represent all actions in a situation and that there may be more than one action required</li> <li>• Add and subtract 2 two-digit whole numbers</li> <li>• Multiply two-digit by one-digit whole numbers</li> <li>• Perform division related to one-digit by one-digit multiplication facts</li> </ul>
	<p>MA.4.M.2.AP.1b Solve one-step real-world problems involving intervals of time in whole numbers using any of the four operations.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent situations using any of the four operations with objects or drawings</li> <li>• Multiply two-digit by one-digit whole numbers</li> <li>• Perform division related to one-digit by one-digit multiplication facts</li> </ul>
<p>MA.4.M.2.2</p>	<p>Solve one- and two-step addition and subtraction real-world problems involving money using decimal notation.</p> <p><b>Access Point</b></p> <p>MA.4.M.2.AP.2 Solve one- and two-step addition and subtraction real-world problems involving money using decimal notation. Sums not to exceed \$0.99 and their related subtraction facts.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” with objects or drawings</li> <li>• Understand the need to represent all actions in a situation and that there may be more than one action required</li> <li>• Add and subtract decimals less than one to the hundredths</li> </ul>

**Geometric Reasoning**

<b>MA.4.GR.1 Draw, classify and measure angles.</b>	
MA.4.GR.1.1	Informally explore angles as an attribute of two-dimensional figures. Identify and classify angles as acute, right, obtuse, straight or reflex.
	<b>Access Point</b> MA.4.GR.1.AP.1 Informally explore angles as an attribute of two-dimensional figures. Limit angles to acute, obtuse, and right.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>MA.4.GR.1.2 Recognize points and lines in two-dimensional figures</li> </ul>
MA.4.GR.1.2	Estimate angle measures. Using a protractor, measure angles in whole-number degrees and draw angles of specified measure in whole-number degrees. Demonstrate that angle measure is additive.
	<b>Access Point</b> MA.4.GR.1.AP.2 Using a tool with a square angle, identify angles as acute, right or obtuse and construct angles that are acute, right or obtuse.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Understand that angles are an attribute of two-dimensional figures</li> <li>Understand the terms "acute," "right," and "obtuse"</li> </ul>
MA.4.GR.1.3	Solve real-world and mathematical problems involving unknown whole-number angle measures. Write an equation to represent the unknown.
	<b>Access Point</b> MA.4.GR.1.AP.3 Recognize that angle measure is additive by exploring when an angle is decomposed into two non-overlapping parts the angle measure of the whole is the sum of the angle measures of the parts.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>Understand that angles are an attribute of two-dimensional figures</li> <li>Recognize that smaller figures can be formed by taking apart larger two-dimensional figures and that larger figures can be formed by putting together smaller two-dimensional</li> </ul>

	figures
<b>MA.4.GR.2 Solve problems involving the perimeter and area of rectangles.</b>	
MA.4.GR.2.1	Solve perimeter and area mathematical and real-world problems, including problems with unknown sides, for rectangles with whole-number side lengths.
	<b>Access Point</b> MA.4.GR.2.AP.1 Solve perimeter and area mathematical and real-world problems for rectangles with given whole-number side lengths.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Distinguish between the concepts of area and perimeter</li> <li>• Find the perimeter of a rectangle with whole-number side lengths</li> <li>• Find the area of a rectangle with whole-number side lengths</li> </ul>
MA.4.GR.2.2	Solve problems involving rectangles with the same perimeter and different areas or with the same area and different perimeters.
	<b>Access Point</b> MA.4.GR.2.AP.2 Explore the relationship between perimeter and area using rectangles with the same perimeter and different areas or with the same area and different perimeters.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Distinguish between the concepts of area and perimeter</li> <li>• Find the perimeter of a rectangle with whole-number side lengths</li> <li>• Find the area of a rectangle with whole-number side lengths</li> </ul>

### **Data Analysis and Probability**

<b>MA.4.DP.1 Collect, represent and interpret data and find the mode, median and range of a data set.</b>	
MA.4.DP.1.1	Collect and represent numerical data, including fractional

	<p>values, using tables, stem-and-leaf plots or line plots.</p> <p><b>Access Point</b></p> <p>MA.4.DP.1.AP.1 Sort and represent numerical data, including fractional values using tables or line plots (when given a scaled number line). Data set to include only whole numbers and halves.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand how data in a table is organized</li> <li>• Understand how to locate values on a horizontal number line that is labeled with whole numbers</li> <li>• Understand that each X or dot on the line plot represents 1 object with that length, temperature, liquid volume, or weight</li> <li>• Recognize two equal parts of a whole as halves</li> <li>• Recognize that mixed numbers represent an amount of wholes and additional parts of a whole</li> </ul>
MA.4.DP.1.2	<p>Determine the mode, median or range to interpret numerical data including fractional values, represented with tables, stem-and-leaf plots or line plots.</p> <p><b>Access Point</b></p> <p>MA.4.DP.1.AP.2 Determine the mode or range to interpret numerical data including fractional values, represented with tables or line plots. Data set to include only whole numbers and halves. Limit the greatest and least number in a data set to a whole number.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand how data in a table is organized</li> <li>• Understand how to locate values on a horizontal number line that is labeled with whole numbers and halves</li> <li>• Understand reading a horizontal number line that is labeled with whole numbers and halves</li> <li>• Understand that each X or dot on the line plot represents 1 object with that length, temperature, liquid volume, or weight</li> <li>• Understand that when identifying the least and greatest measurement value in a data set</li> </ul>



	<p>displayed on a line plot, the location of each measurement value on the number line will be used</p> <ul style="list-style-type: none"> <li>Recognize that mixed numbers represent an amount of wholes and additional parts of a whole</li> </ul>
MA.4.DP.1.3	<p>Solve real-world problems involving numerical data.</p> <p><b>Access Point</b></p> <p>MA.4.DP.1.AP.3 Solve one-step real-world problems involving numerical data represented with tables or line plots. Data set to include only whole numbers and halves. Required operations to involve only the whole number data points in the data set.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand how data in a table is organized</li> <li>Understand reading a horizontal number line that is labeled with whole numbers and halves</li> <li>Understand that each X or dot on the line plot represents 1 object with that length, temperature, liquid volume, or weight</li> <li>Perform grade level Access Point appropriate operations for whole numbers</li> </ul>

**Grade 5 B.E.S.T. Standards Access Points  
Number Sense and Operations**

<b><i>MA.5.NSO.1 Understand the place value of multi-digit numbers with decimals to the thousandths place.</i></b>	
MA.5.NSO.1.1	<p>Express how the value of a digit in a multi-digit number with decimals to the thousandths changes if the digit moves one or more places to the left or right.</p> <p><b>Access Point</b></p> <p>MA.5.NSO.1.AP.1 Explore how the value of a digit in a multi-digit number with decimals to the hundredths changes if the digit moves one place to the left. Multi-digit numbers not to exceed 9.99.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand that 10 hundredths is equal to 1 tenth, and 10 tenths is equal to 1 one</li> <li>Recognize the location of the ones place, the</li> </ul>

	<p>tenths place, and the hundredths place</p> <ul style="list-style-type: none"> <li>• Understand that the digit in the ones place represents the number of ones, the digit in the tenths place represents the number of tenths, and the digit in the hundredths place represents the number of hundredths</li> </ul>
<p>MA.5.NSO.1.2</p>	<p>Read and write multi-digit numbers with decimals to the thousandths using standard form, word form and expanded form.</p> <p><b>Access Point</b></p> <p>MA.5.NSO.1.AP.2 Read and generate multi-digit numbers with decimals to the hundredths using standard form and expanded form. Multi-digit numbers not to exceed 9.99.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Express number names (rote count) up to 100</li> <li>• Identify a number written in standard form when given the name of the number up to 100</li> <li>• Understand that decimals are parts of a whole and that the decimal point separates the whole number values from the decimal values</li> <li>• Understand that the digits in the ones, tenths and hundredths places represent an amount of ones, tenths, and hundredths</li> <li>• Understand that expanded form is the value of the ones, plus the value of the tenths, plus the value of the hundredths. Understand that the numbers 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 refer to 1, 2, 3, 4, 5, 6, 7, 8, 9 tenth(s)</li> <li>• Understand that the numbers 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 refer to 1, 2, 3, 4, 5, 6, 7, 8, 9 hundredth(s)</li> <li>• Generate numbers 0-100 using standard form</li> </ul>
<p>MA.5.NSO.1.3</p>	<p>Compose and decompose multi-digit numbers with decimals to the thousandths in multiple ways using the values of the digits in each place. Demonstrate the compositions or decompositions using objects, drawings and expressions or equations.</p> <p><b>Access Point</b></p> <p>MA.5.NSO.1.AP.3 Compose and decompose multi-digit</p>

	<p>numbers with decimals to the hundredths. Demonstrate each composition or decomposition with objects, drawings, expressions or equations. Multi-digit numbers not to exceed 9.99.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that a group of 10 tenths is equal to 1 whole (e.g., 10 tenth rods is equal to 1 whole flat) and a group of 10 hundredths is equal to 1 tenth (e.g., 10 hundredths cubes is equal to 1 tenth rod)</li> <li>• Represent numbers from the ones place to the hundredths place using ones, tenths, and hundredths</li> <li>• Given a number up to 9.99, understand that the digit in the ones place represents the number of ones, the digit in the tenths place represents the number of tenths, the digit in the hundredths place represents then number of hundredths</li> </ul>
<p>MA.5.NSO.1.4</p>	<p>Plot, order and compare multi-digit numbers with decimals up to the thousandths.</p>
	<p><b>Access Point</b></p> <p>MA.5.NSO.1.AP.4 Plot, order and compare multi-digit numbers with decimals up to the hundredths. Multi-digit numbers not to exceed 9.99.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that a tenth is one-tenth (<math>1/10</math>) of a whole (e.g., if a flat represents 1 whole, then a rod represents a tenth).</li> <li>• Understand that a hundredth is one-hundredth (<math>1/100</math>) of a whole (e.g., if a flat represents 1 whole, then a unit cube represents a hundredth)</li> <li>• Given a number up to 9.99, understand that the digit in the ones place represents the number of ones, the digit in the tenths place represents the number of tenths, the digit in the hundredths place represents then number of hundredths</li> <li>• Use objects to represent numbers up to 9.99 using ones, tenths, and hundredths</li> </ul>

	<ul style="list-style-type: none"> <li>• Use matching of same unit objects (flats, rods, unit cubes) to compare starting with the greatest place value</li> <li>• Understand that <math>&gt;</math> is “greater than”, <math>&lt;</math> is “less than”, and <math>=</math> is “equal to”</li> <li>• Understand that numbers on a number line are plotted in sequential order, numbers that are farther left/lower on the number line have a lesser value, and numbers farther right/higher on the number line have a greater value</li> </ul>
MA.5.NSO.1.5	<p>Round multi-digit numbers with decimals to the thousandths to the nearest hundredth, tenth or whole number.</p> <p><b>Access Point</b></p> <p>MA.5.NSO.1.AP.5 Round multi-digit numbers with decimals to the tenths to the nearest whole number (e.g., 1.7 rounds to 2); and numbers with decimals to the hundredths to the nearest tenth (e.g., 2.36 rounds to 2.4). Multi-digit numbers not to exceed 9.99.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a number up to 9.99, understand that the digit in the ones place represents the number of ones, the digit in the tenths place represents the number of tenths, and the digit in the thousandths place represents the number of thousandths</li> <li>• Understand that numbers on a number line are plotted in sequential order, numbers that are farther left/lower on the number line have a lesser value, and numbers farther right/higher on the number line have a greater value</li> <li>• Plot numbers up to 9.99 on a number line</li> <li>• Identify which whole number or tenth the number being rounded is closest to on a number line</li> <li>• Understand that if the number being rounded is halfway between consecutive whole numbers or tenths then it rounds to the</li> </ul>

	greater whole number or tenth
<b>MA.5.NSO.2 Add, subtract, multiply and divide multi-digit numbers.</b>	
MA.5.NSO.2.1	Multiply multi-digit whole numbers including using a standard algorithm with procedural fluency.
	<b>Access Point</b> MA.5.NSO.2.AP.1 Explore multiplication of two whole numbers, up to two digits by two digit.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod)</li> <li>• Model two-digit numbers using 10s and 1s (e.g., ten-rods and unit cubes)</li> <li>• Represent multiplication situations using objects organized in equal groups or in rectangular arrays and use the representations to find the total</li> </ul>
MA.5.NSO.2.2	Divide multi-digit whole numbers, up to five digits by two digits, including using a standard algorithm with procedural fluency. Represent remainders as fractions.
	<b>Access Point</b> MA.5.NSO.2.AP.2 Apply a strategy to divide two whole numbers up to two digits by one digit including the possibility of whole number remainders.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand that a group of 10 ones is equal to 1 ten (e.g., 10-unit cubes is equal to 1 ten-rod)</li> <li>• Model two-digit numbers using 10s and 1s (e.g., ten-rods and unit cubes)</li> <li>• Represent division expressions (e.g., <math>62 \div 5</math>) using objects or drawings organized in equal groups and use the representations to find the total number of groups or the number in each group</li> </ul>
MA.5.NSO.2.3	Add and subtract multi-digit numbers with decimals to the thousandths, including using a standard algorithm with procedural fluency.

**Access Point**

MA.5.NSO.2.AP.3 Apply a strategy to add and subtract multi-digit numbers with decimals to the tenths (e.g.,  $3.3 + 0.5$ ) and hundredths (e.g.,  $1.25 - 0.12$ ). Multi-digit numbers not to exceed 9.99.

**Essential Understandings:**

- Understand that the digit in the ones place represents the number of ones, the digit in the tenths place represents the number of tenths, and the digit in the hundredths place represents the number of hundredths
- Represent decimals up to 9.99 using ones, tenths, and hundredths
- Understand that a group of 10 tenths is equal to 1 whole and that 1 whole is equal to a group of 10 tenths
- Understand that a group of 10 hundredths is equal to 1 tenth and that 1 tenth is equal to 10 hundredths
- Understand that in adding decimals one adds tenths and tenths and hundredths and hundredths and sometimes it is necessary to compose a tenth and/or a whole
- Understand that in subtracting decimals, one subtracts tenths from tenths and hundredths from hundredths and sometimes it is necessary to decompose a tenth

MA.5.NSO.2.4

Explore the multiplication and division of multi-digit numbers with decimals to the hundredths using estimation, rounding and place value.

**Access Point**

MA.5.NSO.2.AP.4 Explore the estimation of products and quotients of two multi-digit numbers with decimals to the tenths (e.g.,  $8.9 \times 2.3$  becomes  $9 \times 2$  by rounding both factors to the nearest whole number). Multi-digit numbers not to exceed 9.9.

**Essential Understandings:**

- Round multi-digit numbers with decimals to the tenths in an expression to the nearest whole number to create a simpler problem
- Apply a strategy to multiply single digit whole

	<p>numbers and perform the related division facts</p> <ul style="list-style-type: none"> <li>Recognize that rounding multi-digit decimals numbers in an expression prior to multiplying or dividing provides an estimation of a reasonable solution without performing the exact computations required to solve the problem</li> </ul>
MA.5.NSO.2.5	<p>Multiply and divide a multi-digit number with decimals to the tenths by one-tenth and one-hundredth with procedural reliability.</p>
	<p><b>Access Point</b></p> <p>MA.5.NSO.2.AP.5 Explore multiplying and dividing single digit whole numbers by one-tenth and one-hundredth.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand one-tenth can be represented by a rod, and one-hundredth can be represented by a unit cube</li> <li>Represent multiplication situations using objects organized in equal groups and use the representations to find the total</li> <li>Represent division situations using objects organized in equal groups and use the representations to find the number of groups</li> </ul>

## Fractions

<b><i>MA.5.FR.1 Interpret a fraction as an answer to a division problem.</i></b>	
MA.5.FR.1.1	<p>Given a mathematical or real-world problem, represent the division of two whole numbers as a fraction.</p>
	<p><b>Access Point</b></p> <p>MA.5.FR.1.AP.1 Explore the connection between fractions and division in a real-world problem.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand the concept of equal groups. Distinguish between the number of groups, the number in each group, and the total number</li> <li>Represent division situations by organizing objects in equal groups and use the</li> </ul>

	<p>representations to find the number of groups or the number in each group</p> <ul style="list-style-type: none"> <li>• Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described</li> <li>• Partition two-dimensional shapes into equal-sized parts</li> </ul>
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**MA.5.FR.2 Perform operations with fractions.**

MA.5.FR.2.1	<p>Add and subtract fractions with unlike denominators, including mixed numbers and fractions greater than 1, with procedural reliability.</p>
	<p><b>Access Point</b></p> <p>MA.5.FR.2.AP.1a Explore adding and subtracting mixed numbers and fractions greater than 1 with like denominators.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” with objects</li> <li>• Understand fractions in the form of <math>m/n</math> is the result of adding the unit fraction <math>1/n</math> to itself <math>m</math> times</li> <li>• Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described</li> <li>• Understand that the number of equal parts being described is the same as the number of equal parts in the whole, then the fraction is equal to 1</li> </ul>
	<p>MA.5.FR.2.AP.1b Explore adding and subtracting fractions less than one with unlike denominators where one denominator is a multiple of the other (e.g., <math>1/2 + 3/4</math>, <math>2/3 - 1/6</math>).</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition situations involving “adding to” and subtraction situations involving “taking from” with objects</li> <li>• Understand the denominator is the size of the equal parts of the whole and the numerator is the number of equal parts being described</li> </ul>



	<ul style="list-style-type: none"> <li>• Understand that when a whole is partitioned into more parts, the parts are smaller and when a whole is partitioned into less parts, the parts are larger</li> <li>• Understand that a greater quantity of smaller parts can be combined to cover the same area as a lesser quantity of larger parts</li> <li>• Add and subtract fractions less than one with like denominators</li> </ul>
MA.5.FR.2.2	<p>Extend previous understanding of multiplication to multiply a fraction by a fraction, including mixed numbers and fractions greater than 1, with procedural reliability.</p> <p><b>Access Point</b></p> <p>MA.5.FR.2.AP.2 Explore multiplying a unit fraction by a unit fraction.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Multiply a whole number by a unit fraction (e.g., <math>\frac{1}{4} \times 2</math>)</li> <li>• Understand that when multiplying a whole number by a fraction, that the product represents a part of a whole</li> </ul>
MA.5.FR.2.3	<p>When multiplying a given number by a fraction less than 1 or a fraction greater than 1, predict and explain the relative size of the product to the given number without calculating.</p> <p><b>Access Point</b></p> <p>MA.5.FR.2.AP.3 Explore the impact on the size of the product when multiplying a given number by a fraction less than 1 or a whole number.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of equal groups. Distinguish between the number of groups and the number in each group</li> <li>• Recognize whether the number of groups (i.e., the first factor) is more than one whole group, exactly one whole group, or less than one whole group</li> </ul>
MA.5.FR.2.4	<p>Extend previous understanding of division to explore the</p>

	division of a unit fraction by a whole number and a whole number by a unit fraction.
	<b>Access Point</b> MA.5.FR.2.AP.4 Explore the division of a one-digit whole number by a unit fraction. Denominators are limited to 2, 3 or 4.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Represent division situations using objects to find the total number of groups of a given quantity</li> <li>• Recognize that there are 2 halves in one whole, 3 thirds in one whole, and 4 fourths in one whole</li> </ul>

**Algebraic Reasoning**

<b><i>MA.5.AR.1 Solve problems involving the four operations with whole numbers and fractions.</i></b>	
MA.5.AR.1.1	Solve multi-step real-world problems involving any combination of the four operations with whole numbers, including problems in which remainders must be interpreted within the context.
	<b>Access Point</b> MA.5.AR.1.AP.1 Solve one- and two-step real-world problems involving any combination of the four operations with whole numbers. Explore problems in which remainders must be interpreted within the context.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Represent situations involving any combination of the four operations with objects or drawings</li> <li>• Understand the need to represent all actions in a situation and that there may be more than one action required</li> <li>• Add and subtract 2 two-digit whole numbers</li> <li>• Multiply two-digit by one-digit whole numbers</li> <li>• Perform division related to one-digit by one-digit multiplication facts</li> </ul>
MA.5.AR.1.2	Solve real-world problems involving the addition,

	<p>subtraction or multiplication of fractions, including mixed numbers and fractions greater than 1.</p>
	<p><b>Access Point</b>  MA.5.AR.1.AP.2a Solve one-step real-world problems involving addition and subtraction of mixed numbers and fractions greater than one with like denominators.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations with drawings or objects</li> <li>• Apply a strategy to add or subtract mixed numbers and fractions less than one with like denominators</li> </ul>
	<p>MA.5.AR.1.AP.2b Solve one-step real-world problems involving multiplication of unit fractions.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent situations involving multiplication with drawings or objects</li> <li>• Apply a strategy to multiply a unit fraction by a unit fraction</li> </ul>
<p>MA.5.AR.1.3</p>	<p>Solve real-world problems involving division of a unit fraction by a whole number and a whole number by a unit fraction.</p>
	<p><b>Access Point</b>  MA.5.AR.1.AP.3 Solve one-step real-world problems involving division of a whole number by a unit fraction.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent situations involving division with drawings or objects</li> <li>• Apply a strategy to divide a whole number by a unit fraction with denominators limited to 2, 3, or 4</li> </ul>
<p><b><i>MA.5.AR.2 Demonstrate an understanding of equality, the order of operations and equivalent numerical expressions.</i></b></p>	
<p>MA.5.AR.2.1</p>	<p>Translate written real-world and mathematical descriptions into numerical expressions and numerical expressions into written mathematical descriptions.</p>
	<p><b>Access Point</b>  MA.5.AR.2.AP.1 Translate real-world and mathematical</p>

	<p>descriptions into numerical expressions with two terms (e.g., five plus two; the product of three and four).</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand + as a symbol representing the operation of addition and this operation can be indicated by the words "plus" and "sum"</li> <li>• Understand – as a symbol representing the operation of subtraction and this operation can be indicated by the words "minus" and "difference"</li> <li>• Understand × as a symbol representing the operation of multiplication and this operation can be indicated by the words "times" and "product"</li> <li>• Understand ÷ as a symbol representing the operation of division and this operation can be indicated by the words "divided by" and "quotient"</li> </ul>
MA.5.AR.2.2	<p>Evaluate multi-step numerical expressions using order of operations.</p> <p><b>Access Point</b></p> <p>MA.5.AR.2.AP.2 Evaluate an expression containing three terms and one set of parentheses.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that the operation in the parenthesis is performed first</li> <li>• Add and subtract 2 two-digit whole numbers</li> <li>• Multiply two-digit by one-digit whole numbers</li> <li>• Perform division related to one-digit by one-digit multiplication facts</li> </ul>
MA.5.AR.2.3	<p>Determine and explain whether an equation involving any of the four operations is true or false.</p> <p><b>Access Point</b></p> <p>MA.5.AR.2.AP.3 Determine whether an equation (with no more than four terms and up to one set of parentheses) involving any of the four operations with whole numbers is true or false. Limit addition and subtraction to within 100 and limit multiplication and division to the products of two single-digit whole numbers and their related division facts.</p>

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of “equality” as the balance of two values (e.g., if a balance scale is level, then the values are equal and if it is not level, then the values are not equal)</li> <li>• Understand that = is “equal to”</li> <li>• Understand that if the values on either side of the equal sign are the same, then the equation is true and if the values on either side of the equal side are not the same, then the equation is false</li> <li>• Add and subtract 2 two-digit whole numbers</li> <li>• Multiply two-digit by one-digit whole numbers</li> <li>• Perform division related to one-digit by one-digit multiplication facts</li> </ul>
<p>MA.5.AR.2.4</p>	<p>Given a mathematical or real-world context, write an equation involving any of the four operations to determine the unknown whole number with the unknown in any position.</p> <p><b>Access Point</b></p> <p>MA.5.AR.2.AP.4 Given a mathematical or real-world context, generate an equation involving any of the four operations to determine the unknown sum, difference, product or quotient. Sums may not exceed 100 and their related subtraction facts. Multiplication and division may not exceed two-digit by one-digit.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand + as a symbol representing the operation of addition and – as a symbol representing the operation of subtraction</li> <li>• Understand × as a symbol representing the operation of multiplication and ÷ as a symbol representing the operation of division</li> <li>• Understand = as a symbol representing the equality of two values</li> <li>• Understand a symbol (e.g., ___ or □) may be used to represent an unknown number in an equation</li> <li>• Interpret relevant information in a real-world context</li> </ul>

	<ul style="list-style-type: none"> <li>• Find the sum or differences of 2 two-digit whole numbers</li> <li>• Find products or quotients of two-digit by one-digit whole numbers</li> </ul>
<b>MA.5.AR.3 Analyze patterns and relationships between inputs and outputs.</b>	
MA.5.AR.3.1	Given a numerical pattern, identify and write a rule that can describe the pattern as an expression.
	<b>Access Point</b> MA.5.AR.3.AP.1 Given a numerical pattern, identify a one-step rule that can describe the pattern.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand that patterns are repeated and predictable and can be described using a rule</li> <li>• Perform basic operations</li> </ul>
MA.5.AR.3.2	Given a rule for a numerical pattern, use a two-column table to record the inputs and outputs.
	<b>Access Point</b> MA.5.AR.3.AP.2 Given the inputs and a one-step addition or subtraction rule for a numerical pattern, use a two-column table to record the outputs.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand how data in a table is organized</li> <li>• Understand that patterns are repeated and predictable and can be extended by following a rule</li> <li>• Find the sum or differences of up to 2 two-digit whole numbers</li> </ul>

### Measurement

<b>MA.5.M.1 Convert measurement units to solve multi-step problems.</b>	
MA.5.M.1.1	Solve multi-step real-world problems that involve converting measurement units to equivalent measurements within a single system of measurement.
	<b>Access Point</b> MA.5.M.1.AP.1a Using a conversion sheet, convert within a

single system of measurement using the units: miles, yards, feet, inches; pounds, ounces; gallons, quarts, pints, cups; and hours, minutes. Only whole number measurements may be used.

**EU**

- Understand the relationship between the size of units of measurements within the same system of units (e.g., miles are longer than inches; ounces are lighter than pounds; gallons hold more than a pint; minutes are shorter than hours)
- Understand that a larger unit of measurement can be converted to a smaller unit of measurement within a single system of measurement and as a result of the conversion there will be a greater number of the smaller unit (e.g., when converting feet to inches there will be a greater number of inches since inches is a smaller unit than feet; 2 feet is equal to 24 inches)
- Understand that a smaller unit of measurement can be converted to a larger unit of measurement within a single system of measurement and as a result of the conversion there will be a smaller number of the larger unit (e.g., when converting inches to feet there will be a smaller number of feet since feet is a larger unit than inches; 24 inches is equal to 2 feet)

MA.5.M.1.AP.1b Using a conversion sheet, solve one- and two-step real-world problems that involve converting measurement units (i.e., miles, yards, feet, inches; pounds, ounces; gallons, quarts, pints, cups; and hours, minutes) to equivalent measurements within a single system of measurement. Only whole number measurements may be used.

**EU**

- Understand the relationship between the size of units of measurements within the same system of units (e.g., yards are longer than inches; pounds are heavier than ounces; gallons hold more than a pint; hours are

	<p>longer than minutes)</p> <ul style="list-style-type: none"> <li>• Understand that a larger unit of measurement can be converted to a smaller unit of measurement within a single system of measurement and as a result of the conversion there will be a greater number of the smaller unit (e.g., when converting feet to inches there will be a greater number of inches since inches is a smaller unit than feet; 2 feet is equal to 24 inches)</li> <li>• Represent situations involving any combination of the four operations with objects or drawings</li> <li>• Understand the need to represent all actions in a situation and that there may be more than one action required</li> <li>• Add and subtract 2 two-digit whole numbers</li> <li>• Multiply two-digit by one-digit whole numbers</li> <li>• Perform division related to one-digit by one-digit multiplication facts</li> </ul>
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***MA.5.M.2 Solve problems involving money.***

MA.5.M.2.1	Solve multi-step real-world problems involving money using decimal notation.
	<p><b>Access Point</b></p> <p>MA.5.M.2.AP.1 Solve one- and two-step addition and subtraction real-world problems involving money using decimal notation with all terms less than \$20.00 (e.g., \$11.74 + \$5.31, \$10.99 - \$3.26).</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Represent addition and subtraction situations involving “adding to” and “taking from” with objects or drawings</li> <li>• Understand the need to represent all actions in a situation and that there may be more than one action required</li> <li>• Add and subtract multi-digit numbers (with all terms less than 20.00) with decimals to the hundredths</li> </ul>

**Geometric Reasoning**



**MA.5.GR.1 Classify two-dimensional figures and three-dimensional figures based on defining attributes.**

MA.5.GR.1.1

Classify triangles or quadrilaterals into different categories based on shared defining attributes. Explain why a triangle or quadrilateral would or would not belong to a category.

**Access Point**

MA.5.GR.1.AP.1a Sort triangles into different categories based on the size of their angles. Triangles include acute, obtuse, and right.

**Essential Understandings:**

- Understand that angles are attributes of two-dimensional figures
- Using a tool with a square angle, identify angles as acute, right, or obtuse

MA.5.GR.1.AP.1b Sort quadrilaterals into different categories based on shared defining attributes. Explore why a quadrilateral would or would not belong to a category. Quadrilaterals include parallelograms, rhombi, rectangles, squares and trapezoids.

**Essential Understandings:**

- Identify specified defining attributes (i.e., parallel sides, equal sides, right angles, acute angles, obtuse angles) in isolated quadrilaterals

MA.5.GR.1.2

Identify and classify three-dimensional figures into categories based on their defining attributes. Figures are limited to right pyramids, right prisms, right circular cylinders, right circular cones and spheres.

**Access Point**

MA.5.GR.1.AP.2 Identify and sort three-dimensional figures into categories based on their defining attributes. Figures are limited to right rectangular pyramids, right rectangular prisms, right circular cylinders, right circular cones and spheres.

**Essential Understandings:**

- Identify specified defining attributes (i.e., faces, bases, edges, curved surface, vertices, point) in isolated three-dimensional figures.
- Understand the defining attributes of "right rectangular pyramids," "right rectangular

prisms," "right circular cylinders," "right circular cones," and "spheres"

***MA.5.GR.2 Find the perimeter and area of rectangles with fractional or decimal side lengths.***

MA.5.GR.2.1

Find the perimeter and area of a rectangle with fractional or decimal side lengths using visual models and formulas.

**Access Point**

MA.5.GR.2.AP.1 Find the perimeter and area of a rectangle with decimal side lengths using a visual model and calculator.

**Essential Understandings:**

- Distinguish between the concepts of area and perimeter
- Find the perimeter of a rectangle with whole-number side lengths by adding the lengths of the sides
- Find the area of a rectangle with whole-number side lengths by multiplying the side lengths
- Understand how to use a calculator to perform basic mathematical operations with whole numbers

***MA.5.GR.3 Solve problems involving the volume of right rectangular prisms.***

MA.5.GR.3.1

Explore volume as an attribute of three-dimensional figures by packing them with unit cubes without gaps. Find the volume of a right rectangular prism with whole-number side lengths by counting unit cubes.

**Access Point**

MA.5.GR.3.AP.1 Explore volume as an attribute of three-dimensional figures that can be measured by packing them with unit cubes without gaps.

**Essential Understandings:**

- Understand area as an attribute of a two-dimensional figure that can be measured by covering the figure with unit squares without gaps or overlaps
- Recognize the difference between a two- and

	three-dimensional figure
MA.5.GR.3.2	Find the volume of a right rectangular prism with whole-number side lengths using a visual model and a formula.
	<b>Access Point</b> MA.5.GR.3.AP.2 Find the volume of a right rectangular prism with whole-number side lengths by counting unit cubes. Explore that the volume is the same as what would be found by multiplying the edge lengths.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand the concept of volume</li> <li>• Identify the base and understand the concept of multiplication using arrays to find the area of the base.</li> <li>• Identify the height as the number of layers</li> </ul>
MA.5.GR.3.3	Solve real-world problems involving the volume of right rectangular prisms, including problems with an unknown edge length, with whole-number edge lengths using a visual model or a formula. Write an equation with a variable for the unknown to represent the problem.
	<b>Access Point</b> MA.5.GR.3.AP.3 Solve real-world problems involving the volume of right rectangular prisms with given whole-number edge lengths using a visual model or formula.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand the concept of volume</li> <li>• Find the volume of a right rectangular prism with whole-number edge lengths by counting unit cubes</li> <li>• Multiply three single digit numbers</li> </ul>
<b><i>MA.5.GR.4 Plot points and represent problems on the coordinate plane.</i></b>	
MA.5.GR.4.1	Identify the origin and axes in the coordinate system. Plot and label ordered pairs in the first quadrant of the coordinate plane.
	<b>Access Point</b> MA.5.GR.4.AP.1 Explore the first quadrant of the coordinate plane including the origin, axes and points located by using ordered pairs.

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize points and lines</li> <li>• Locate numbers on a number line</li> </ul>
MA.5.GR.4.2	<p>Represent mathematical and real-world problems by plotting points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.</p> <p><b>Access Point</b></p> <p>MA.5.GR.4.AP.2 Plot and label ordered pairs in the first quadrant of the coordinate plane.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the origin, axes and points located by using ordered pairs</li> <li>• Locate numbers on a number line</li> </ul>

### **Data Analysis and Probability**

<b><i>MA.5.DP.1 Collect, represent and interpret data and find the mean, mode, median or range of a data set.</i></b>	
MA.5.DP.1.1	<p>Collect and represent numerical data, including fractional and decimal values, using tables, line graphs or line plots.</p> <p><b>Access Point</b></p> <p>MA.5.DP.1.AP.1 Sort and represent numerical data, including fractional values using tables or line plots (when given a scaled number line). Data set to include only whole numbers, halves and quarters.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand how data in a table is organized</li> <li>• Understand how to locate values on a horizontal number line that is labeled with whole numbers and halves</li> <li>• Understand that each X or dot on the line plot represents 1 object with that length, temperature, liquid volume, or weight</li> <li>• Recognize two equal parts of a whole as halves</li> <li>• Recognize four equal parts of a whole as fourths or quarters</li> <li>• Recognize that mixed numbers represent an</li> </ul>

	amount of wholes and additional parts of a whole
MA.5.DP.1.2	Interpret numerical data, with whole-number values, represented with tables or line plots by determining the mean, mode, median or range.
	<b>Access Point</b> MA.5.DP.1.AP.2 Interpret numerical data, with whole-number values, represented with tables or line plots by determining the mean, mode or range. Line plot scales to include only whole numbers, halves and quarters.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand how data in a table is organized</li> <li>• Understand reading a horizontal number line that is labeled with whole numbers, halves, and quarters</li> <li>• Understand that each X or dot on the line plot represents 1 object with that length, temperature, liquid volume, or weight</li> <li>• Understand that when identifying the least and greatest measurement value in a data set displayed on a line plot, the location of each measurement value on the number line will be used</li> <li>• Perform grade level Access Point appropriate subtraction of whole numbers</li> <li>• Find the sum (up to 99) of multiple addends</li> <li>• Divide two-digit numbers by one digit with no remainders</li> </ul>

**Grade 6  
Number Sense and Operations**

<b><i>MA.6.NSO.1 Extend knowledge of numbers to negative numbers and develop an understanding of absolute value.</i></b>	
MA.6.NSO.1.1	Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.
	<b>Access Point</b> MA.6.NSO.1.AP.1 Plot, order, and compare rational numbers (positive and negative integers within 10 from 0,

fractions with common denominators, decimals up to the hundredths and percentages) in the same form.

**Essential Understandings:**

- Interactive number lines with positive and negative numbers
- Use manipulatives to support students in comparing the size of rational numbers
- Label number lines
- Label points on a number line
- Use manipulatives on the number line to identify the number with the greatest value by determining which number is furthest to the right on the number line
- Use manipulatives (fractions bars, base ten blocks, etc.) to determine the relative size of fractions and decimals
- Virtual manipulatives for online instruction
- Use inequality symbols ( $<$ ,  $>$ , or  $=$ ) to label which number has the greatest value
- Vocabulary: integers, numerators, denominators, decimal place value (tenths, hundredths, thousandths), positive & negative numbers

MA.6.NSO.1.2

Given a mathematical or real-world context, represent quantities that have opposite direction using rational numbers. Compare them on a number line and explain the meaning of zero within its context.

**Access Point**

MA.6.NSO.1.AP.2 Represent positive and negative numbers in the same form on a number line given a real-world situation and explain the meaning of zero within its context.

**Essential Understandings:**

- Describe negative numbers as numbers less than zero
- Understand less/same/more in context (e.g., temperature, ground level)
- Use vertical number lines, in addition to horizontal number lines, to illustrate negative numbers

	<ul style="list-style-type: none"> <li>• Select pictorial representations of less than zero in the real-world scenarios</li> <li>• Understand the meaning of zero and where it falls on the number line</li> <li>• Recognize that on a number line all the numbers to the right of zero are positive and all the numbers to the left of zero are negative</li> <li>• Recognize that negative numbers have a negative symbol (-) before the number</li> <li>• Recognize that positive numbers either have a (+) symbol or no symbol before the number</li> <li>• Vocabulary: integers, numerators, denominators, decimal place value (tenths, hundredths, thousandths), positive &amp; negative numbers</li> </ul>
MA.6.NSO.1.3	<p>Given a mathematical or real-world context, interpret the absolute value of a number as the distance from zero on a number line. Find the absolute value of rational numbers.</p> <p><b>Access Point</b></p> <p>MA.6.NSO.1.AP.3 Find the meaning of absolute value using the numbers -30 to 30.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use counting to support students in determining the distance from zero to the selected number value on the number line</li> <li>• Use a placeholder or manipulatives to support students in determining the value to count to when determining the distance from zero</li> <li>• Define absolute value</li> <li>• Identify the value of the number and the distance of that number from zero on a number line</li> <li>• Match the positive and the negative value of the same number on the number line</li> <li>• Identify absolute values of numbers</li> <li>• Identify absolute value symbols, i.e.,  -5 </li> <li>• Vocabulary: positive &amp; negative numbers, absolute value, distance from zero, zero</li> </ul>
MA.6.NSO.1.4	Solve mathematical and real-world problems involving

	<p>absolute value, including the comparison of absolute value.</p> <p><b>Access Point</b></p> <p>MA.6.NSO.1.AP.4 Use manipulative, models or tools to compare absolute value in mathematical and real-world problems.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use manipulatives, like number lines or playing cards, to compare quantities</li> <li>• Model distance from zero to compare relative size of the quantities</li> <li>• Identify the value of the number and the distance of that number from zero on a number line</li> <li>• Identify absolute values of numbers</li> <li>• Identify absolute value symbols, i.e., <math> -5 </math></li> <li>• Create a life-size number line on the classroom floor for the students to practice walking the distance from zero</li> <li>• Create individual number lines on student desks</li> <li>• Vocabulary: absolute value, positive and negative numbers, zero, compare</li> </ul>
<p><b><i>MA.6.NSO.2 Add, subtract, multiply and divide positive rational numbers.</i></b></p>	
<p>MA.6.NSO.2.1</p>	<p>Multiply and divide positive multi-digit numbers with decimals to the thousandths, including using a standard algorithm with procedural fluency.</p> <p><b>Access Point</b></p> <p>MA.6.NSO.2.AP.1 Solve one-step multiplication and division problems involving positive decimals whose place value ranges from the tens to the hundredths places.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand and apply the concepts of multiplication and division</li> <li>• Identify a decimal</li> <li>• Given a context, choose the correct operation</li> <li>• Relate decimals to pictorial representations</li> <li>• Create an array of objects into groups to model the role of equal groups in a</li> </ul>



	<p>multiplication or division situation</p> <ul style="list-style-type: none"> <li>• Create a pictorial array for the mathematical equation following multiplication or division rules for an equation</li> <li>• Understanding the steps of the standard algorithm for multiplication and division - See images below for examples of the standard algorithm as a visual model and numerical process</li> <li>• Understand the following symbols, <b>concepts</b>, and vocabulary: place value, +, -, ×, ÷, fractions, decimal (<math>a/b</math>, <math>a</math>).</li> <li>• Use mnemonic devices to help students remember the process for Division Family to assist with algorithm: Dad (divide), Mom (multiply), Sister (subtraction), Brother (bring down), and Rover (repeat or remainder)</li> <li>• Math tools: <ul style="list-style-type: none"> <li>• Base Ten Manipulatives</li> <li>• Tables and Graphic organizers</li> <li>• Calculator</li> <li>• Place Value Table</li> <li>• Multiplication and Division Tables</li> <li>• Division template</li> </ul> </li> </ul>
MA.6.NSO.2.2	<p>Extend previous understanding of multiplication and division to compute products and quotients of positive fractions by positive fractions, including mixed numbers, with procedural fluency.</p> <p><b>Access Point</b></p> <p>MA.6.NSO.2.AP.2 Use tools to calculate the product and quotient of positive fractions by positive fractions, including mixed numbers, using the standard algorithms.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use a template to support modeling using the standard algorithm</li> <li>• Construct a template for multiplying fractions</li> <li>• Example for standard algorithm for multiplying fractions</li> <li>• Anchor chart for multiplying mixed numbers</li> </ul>

Multiplying Mixed Number Using the Standard Algorithm

$$1\frac{+1}{x2} \times 4\frac{+1}{x2}$$

1) First convert each factor into an improper fraction.

$$\frac{3}{2} \times \frac{9}{2}$$

$$\frac{3 \times 9}{2 \times 2} = \frac{27}{4}$$

2) Multiply the numerators and denominators to get the product.

$$\frac{27}{4} \begin{array}{r} 4 \overline{)27} \\ \underline{-24} \\ 3 \end{array} = 4\frac{3}{4}$$

3) Divide the denominator into the numerator to get the mixed number.

- Understand the following symbols, concepts, and vocabulary: product, quotient, fraction, mixed numbers, math symbols +, ÷ - x, =

MA.6.NSO.2.3

Solve multi-step real-world problems involving any of the four operations with positive multi-digit decimals or positive fractions, including mixed numbers.

**Access Point**

MA.6.NSO.2.AP.3a Solve one-step real-world problems involving any of the four operations with positive decimals ranging from the thousand to thousandth place value.

**Essential Understandings:**

- Understand and apply the concepts of addition, subtraction, multiplication, and division
- Identify a fraction and decimal
- Given a context, choose the correct operation (e.g., altogether, take away).
- Relate fractions and decimals to pictorial representations
- Understand the following symbols, concepts, and vocabulary: +, -, ×, ÷, fraction, decimal (a/b, a), place value

MA.6.NSO.2.AP.3b Solve one-step real-world problems involving any of the four operations with positive fractions and mixed numbers with like denominators.

**Essential Understandings:**

- Understand and apply the concepts of addition, subtraction, multiplication, and division
- Identify a fraction and decimal

- Given a context, choose the correct operation (e.g., altogether, take away)
- Relate fractions and decimals to pictorial representations.
- Understand the meaning of “fractions greater than 1” (Note: replaces the language of “improper fractions”)
- Understand converting mixed numbers into fractions greater than 1
- Understand the following symbols, concepts, and vocabulary: +, -, ×, ÷, fraction, decimal (a/b, a), mixed number
- Use Anchor charts to support modeling multiplying mixed numbers

Multiplying Mixed Number Using the Standard Algorithm

$$1\frac{+1}{x2} \times 4\frac{+1}{x2}$$

1) First convert each factor into an improper fraction.

$$\frac{3}{2} \times \frac{9}{2}$$

2) Multiply the numerators and denominators to get the product.

$$\frac{3 \times 9}{2 \times 2} = \frac{27}{4}$$

$$\frac{27}{4} = 4\frac{4}{4} = 4\frac{3}{4}$$

3) Divide the denominator into the numerator to get the mixed number.

**MA.6.NSO.3 Apply properties of operations to rewrite numbers in equivalent forms.**

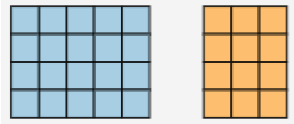
MA.6.NSO.3.1 Given a mathematical or real-world context, find the greatest common factor and least common multiple of two whole numbers.

**Access Point**

MA.6.NSO.3.AP.1 Use tools to find the greatest common factor and least common multiple of two whole numbers under 50.

**Essential Understandings:**

- Using manipulatives to separate two given sets into the largest possible evenly divided groups, with each group containing the same number (e.g., 12 manipulatives will have three groups of four; eight manipulatives will have two groups of four)
- Identify multiples of whole numbers using a

	<p>hundreds chart or multiplication table with markers</p> <ul style="list-style-type: none"> <li>• Identify factors of whole numbers using a hundreds chart or multiplication table with markers</li> <li>• Understand related vocabulary (factor, multiple, least, common)</li> </ul>
<p>MA.6.NSO.3.2</p>	<p>Rewrite the sum of two composite whole numbers having a common factor, as a common factor multiplied by the sum of two whole numbers.</p> <p><b>Access Point</b></p> <p>MA.6.NSO.3.AP.2 Use the distributive property to express a number as the sum of two whole numbers multiplied by a common factor.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Apply the concepts of addition, subtraction, multiplication, and division using manipulatives</li> <li>• Use base ten blocks to represent the numbers in the distributive property (for example, 4 (9 + 2) would look like nine blocks plus two blocks repeated four times)</li> </ul> <p><math>20 + 12 = 4 (5 + 3)</math></p>  <ul style="list-style-type: none"> <li>• Draw a picture representing the distributive property</li> <li>• Use the distributive property to write simple expressions using area</li> <li>• Understand that when using the distributive property, all the numbers inside the parentheses are multiplied by the number outside the parentheses</li> </ul>
<p>MA.6.NSO.3.3</p>	<p>Evaluate positive rational numbers and integers with natural number exponents.</p> <p><b>Access Point</b></p> <p>MA.6.NSO.3.AP.3a Identify what an exponent represents</p>

(e.g.,  $8^3 = 8 \times 8 \times 8$ ).

**Essential Understandings:**

- Produce the correct amount of base numbers to be multiplied given a graphic organizer or template
- Select the correct expanded form of what an exponent represents (e.g.,  $8^3 = 8 \times 8 \times 8$ )
- Identify the number of times the base number will be multiplied based on the exponent
- Understand the following concepts, symbols, and vocabulary: base number, exponent

MA.6.NSO.3.AP.3b Solve numerical expressions involving whole-number bases and exponents (e.g.,  $5 + 2^4 \times 6 = 101$ ).

**Essential Understandings:**

- Apply the concepts of addition, subtraction, multiplication, and division
- Draw a picture or use manipulatives to understand the different parts of an expression
- Solve a numerical expression with whole numbers using tools, as needed (i.e., calculator, multiplication chart)
- Locate an exponent in an expression.
- Understand the following concepts, symbols, and vocabulary for exponent

MA.6.NSO.3.4

Express composite whole numbers as a product of prime factors with natural number exponents.

**Access Point**

MA.6.NSO.3.AP.4 Use a tool to show the prime factors of a number (e.g.,  $20 = 2 \times 2 \times 5$ ).

**Essential Understandings:**

- Use tools (such as, Multiplication Chart, Calculator, Prime Factorization Calculator) to identify factors
- Understand and use divisibility rules to find factors
- Understand and use factor trees to illustrate prime factorization  $48 = 2 * 2 * 2 * 2 * 3$

	48			
	/ \			
2		24		
		/ \		
	2		12	
			/ \	
		2		6
				/ \
			2	3

- Identify or list the factors of a numbers
- Identify whether a number is prime or composite
- Vocabulary: Prime number, composite number, factor, multiple, divisible, factorization

MA.6.NSO.3.5

Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.

**Access Point**

MA.6.NSO.3.AP.5 Rewrite a number 3 or less, as a fraction, decimal or a percent.

**Essential Understandings:**

- Use models or manipulatives to support students in converting between forms
- Use tools to support students in understanding equivalent forms (i.e., a calculator to convert from a fraction to a decimal or a hundred-grid chart to model converting from a decimal to a percent)
- Use anchor charts to support students in remembering the process for converting between number forms (fractions, decimals, and percent)
- Vocabulary: positive rational number, fraction, terminating decimal, percent, equivalent, conversion, mixed number

**MA.6.NSO.4 Extend understanding of operations with integers.**

MA.6.NSO.4.1

Apply and extend previous understandings of operations with whole numbers to add and subtract integers with procedural fluency.

	<p><b>Access Point</b> MA.6.NSO.4.AP.1 Use tools to Add and subtract integers between 50 and -50.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use number lines to illustrate addition and subtraction with integers</li> <li>• Use both vertical and horizontal number lines to perform operations</li> <li>• Using manipulatives to assist adding and subtracting integers</li> <li>• Create life-size number line to model the adding and subtracting with integers</li> <li>• Use Anchor charts to help students to determine the sign of the answer when adding and subtracting integers</li> <li>• Use visual displays to illustrate why the signs of the answers may be positive or negative</li> <li>• Understand the following concepts, symbols, and vocabulary: positive and negative numbers, integers, math symbols -, +, =</li> </ul>
MA.6.NSO.4.2	<p>Apply and extend previous understandings of operations with whole numbers to multiply and divide integers with procedural fluency.</p> <p><b>Access Point</b> MA.6.NSO.4.AP.2 Use tools to multiply and divide integers between 20 and -20.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use manipulatives (two color counters) to support multiplying and dividing integers</li> <li>• Use multiplication table to support multiplication and division</li> <li>• Use Anchor charts to help students find the sign of the answer to the problem</li> <li>• Use visual displays to illustrate why the signs of the answers may be positive or negative</li> <li>• Understand the following concepts, symbols, and vocabulary: positive and negative numbers, integers, math symbols <math>\times</math>, <math>\div</math>, =</li> </ul>
<p><b>MA.6.AR.1 Apply previous understanding of arithmetic expressions</b></p>	

**to algebraic expressions.**

MA.6.AR.1.1	<p>Given a mathematical or real-world context, translate written descriptions into algebraic expressions and translate algebraic expressions into written descriptions.</p> <p><b>Access Point</b></p> <p>MA.6.AR.1.AP.1 Write or select an algebraic expression that represents a real-world situation.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"><li>• Use manipulatives to represent a situation (i.e., John has five apples, and he gives some to Jim = <math>5 - x</math>)</li><li>• When given a verbal expression (i.e., eight plus <math>y</math>), students must select the appropriate algebraic expression (<math>8 + y</math>)</li><li>• Identify key words that signal operations to support students in recognizing operations in word problems</li><li>• Use math tools like a graphic organizer or manipulatives to support students in illustrating the expression before selecting or writing from a real-world situation</li><li>• Understand the following concepts, symbols, and vocabulary: expression, math symbols <math>+</math>, <math>-</math>, <math>x</math>, <math>\div</math>, <math>=</math>, terms and like terms</li></ul>
MA.6.AR.1.2	<p>Translate a real-world written description into an algebraic inequality in the form of <math>xx &gt; oo</math>, <math>xx &lt; oo</math>, <math>xx \geq oo</math> or <math>xx \leq oo</math>. Represent the inequality on a number line.</p> <p><b>Access Point</b></p> <p>MA.6.AR.1.AP.2 Write or select an inequality that represents a real-world situation.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"><li>• Use objects to compare whole numbers</li><li>• Use objects to represent inequalities with whole numbers</li><li>• Use pictures/tables to represent inequalities with whole numbers (i.e., input/output chart or graphic organizer)</li><li>• Understand the difference between a true and a false mathematical statement</li><li>• Use tools, like number lines and</li></ul>



	<p>manipulatives, to support students in identifying appropriate inequality statements</p> <ul style="list-style-type: none"> <li>• Understand the following concept and vocabulary of inequality</li> <li>• Understand the following symbols <math>+</math>, <math>-</math>, <math>\div</math>, <math>=</math>, <math>\times</math>, <math>&lt;</math>, <math>&gt;</math>, <math>\neq</math>, <math>\leq</math>, <math>\geq</math></li> </ul>
<p>MA.6.AR.1.3</p>	<p>Evaluate algebraic expressions using substitution and order of operations.</p> <p><b>Access Point</b></p> <p>MA.6.AR.1.AP.3 Solve an expression using substitution with no more than two operations.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Evaluate an expression using substitution with manipulatives (e.g., find the value of <math>x + 4</math> when <math>x = 2</math> using manipulatives)</li> <li>• Use tools or objects to solve expressions with whole numbers</li> <li>• Use tools or models, like an input/output table or number line, to solve expressions using substitution</li> <li>• Understand the following concepts, symbols, and vocabulary: expression, substitution, operations, input, output, set, simplify, variable, math symbols <math>+</math>, <math>-</math>, <math>\div</math>, <math>\times</math></li> </ul>
<p>MA.6AR.1.4</p>	<p>Apply the properties of operations to generate equivalent algebraic expressions with integer coefficients.</p> <p><b>Access Point</b></p> <p>MA.6.AR.1.AP.4 Use tools or models to combine like terms in an expression with no more than 4 operations.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use manipulatives or visual model to combine like terms (i.e., demonstrate <math>5x + 3x</math> by combining 5 blue blocks and 3 blue blocks)</li> <li>• Identify what is and what is not a like term (e.g., 3 oranges and 5 apples are not like terms; 3 oranges and 5 oranges are like terms)</li> <li>• Use tools, as needed, to complete the four operations with integers such as number lines,</li> </ul>

	<p>calculators, counters, algebra tiles interactive whiteboards, T-tables</p> <ul style="list-style-type: none"> <li>• Explicitly teach strategies for determining the operation required to solve a single step problem</li> <li>• Use adding and subtracting strategies to combine like terms</li> <li>• Use arrays or input/output tables to model substitution</li> <li>• Use a template for simplifying an expression</li> <li>• Use grids or graphic organizers to create arrays</li> <li>• Understand the following concepts, symbols, and vocabulary: like terms, combine, variables, expression, positive integer, negative integer, math symbols +, -, x, ÷</li> </ul>
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***MA.6.AR.2 Develop an understanding for solving equations and inequalities. Write and solve one-step equations in one variable.***

MA.6.AR.2.1	<p>Given an equation or inequality and a specified set of integer values, determine which values make the equation or inequality true or false.</p>
	<p><b>Access Point</b></p> <p>MA.6.AR.2.AP.1 Choose which values, from a set of 5 or fewer integers, make an equation or inequality true.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Determine which of the following values make the inequality <math>x + 1 &lt; 2</math> true: -4, -2, 0, 1</li> <li>• Understand the difference between a true and a false mathematical statement</li> <li>• Evaluate an equations or inequality using substitution with manipulatives (e.g., find the value of <math>x + 4</math> when <math>x = 2</math> using manipulatives)</li> <li>• Use tools or objects to solve equations or inequalities with whole numbers</li> <li>• Use tools or models, like an input/output table or number line, to solve equations or inequalities using substitution</li> <li>• Understand the following concepts, symbols,</li> </ul>

	<p>and vocabulary: expression, substitution, operations, input, output, set, simplify, variable, math symbols <math>+</math>, <math>-</math>, <math>\div</math>, <math>\times</math>, <math>&lt;</math>, <math>&gt;</math>, <math>=</math>, <math>\leq</math>, <math>\geq</math></p>
<p>MA.6.AR.2.2</p>	<p>Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.</p> <p><b>Access Point</b>  MA.6.AR.2.AP.2 Solve real world, one-step linear equations using addition and subtraction involving integers.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Solve one-step equations with the variable on the left side and right side of the equation</li> <li>• Use objects to solve one-step addition and subtraction equations with integers</li> <li>• Use objects to solve one-step addition and subtraction equations with whole numbers</li> <li>• Match a representation of an equation with a variable to a real-world problem.</li> <li>• Use a model to illustrate properties of equality by setting up an equation in which both sides are equal. (For example: <math>x + 4 = 9</math>; <math>x + 4 - 4 = 9 - 4</math>)</li> <li>• Understand the following concepts, symbols, and vocabulary: variable, integer, solution, equation, <math>+</math>, <math>-</math>, <math>=</math></li> </ul>
<p>MA.6.AR.2.3</p>	<p>Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.</p> <p><b>Access Point</b>  MA.6.AR.2.AP.3 Solve real world, one-step linear equations using multiplication and division involving integers.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Solve one-step equations with the variable on the left side and right side of the equation</li> <li>• Use objects to solve one-step multiplication and division equations with integers</li> </ul>

	<ul style="list-style-type: none"> <li>• Use objects to solve one-step multiplication and division equations with whole numbers</li> <li>• Match a representation of an equation with a variable to a real-world problem</li> <li>• Use a model to illustrate properties of equality by setting up an equation in which both sides are equal (for example: <math>4x = 12</math>; <math>4x \div 4 = 12 \div 4</math>)</li> <li>• Understand the following concepts, symbols, and vocabulary: variable, integer, solution, equation, <math>\times</math>, <math>\div</math>, <math>=</math></li> </ul>
MA.6.AR.2.4	<p>Determine the unknown decimal or fraction in an equation involving any of the four operations, relating three numbers, with the unknown in any position.</p> <p><b>Access Point</b></p> <p>MA.6.AR.2.AP.4 Solve a one-step equation using fractions with like denominators or decimals with place value ranging from the thousand to the thousandths.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Solve one-step equations with the variable on the left side and right side of the equation</li> <li>• Use objects to solve one-step equations with fractions with like denominators using all four operations</li> <li>• Use objects to solve one-step equations with decimals using all four operations</li> <li>• Use tools or models to solve one-step equation using fractions or decimals</li> <li>• Use a model to illustrate properties of equality by setting up an equation in which both sides are equal (for example: <math>4x = 1.2</math>; <math>4x \div 4 = 1.2 \div 4</math>)</li> <li>• Understand the following concepts, symbols, and vocabulary: variable, fraction, decimal, solution, equation, <math>+</math>, <math>-</math>, <math>\times</math>, <math>\div</math>, <math>=</math></li> </ul>
<b>MA.6.AR.3 Understand ratio and unit rate concepts and use them to solve problems.</b>	
MA.6.AR.3.1	Given a real-world context, write and interpret ratios to show the relative sizes of two quantities using appropriate

	<p>notation: <math>aa, oo</math> to <math>bb</math>, or <math>oo: bb</math> where <math>bb \neq 0</math></p> <p><b>Access Point</b></p> <p>MA.6.AR.3.AP.1 Given a real-world context, write and interpret ratios to show the relative sizes of two quantities using notation: <math>a/b</math>, <math>a</math> to <math>b</math>, or <math>a:b</math> where <math>b \neq 0</math> with guidance and support.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use a template to support students in writing a ratio using notation</li> <li>• Write the same ratio relationship using different notation</li> <li>• Given a scenario, students can use manipulatives to represent the ratio relationship. (e.g., Each person wants 2 eggs and there are 4 people, how many eggs do you need for the whole group?)</li> <li>• Use data presented in tables or graphs and manipulatives to answer questions about ratios</li> <li>• Given a scenario, students can create a ratio relationship using appropriate notation. (e.g., Each person wants 2 eggs and there are 4 people, how many eggs do you need for the whole group? Answer: 8 eggs: 4 people)</li> <li>• Understand the following concepts, symbols, and vocabulary: ratio, relationship, notation, per, each, <math>:</math>, <math>/</math>, to</li> </ul>
MA.6.AR.3.2	<p>Given a real-world context, determine a rate for a ratio of quantities with different units. Calculate and interpret the corresponding unit rate</p> <p><b>Access Point</b></p> <p>MA.6.AR.3.AP.2 Given a rate, calculate the unit rate for a ratio with different units.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Enter data into a T-chart to demonstrate unit rate</li> <li>• Use a table with visuals or objects to answer questions about a unit rate</li> <li>• Use or create visual images to illustrate the ratio from a word problem or verbal</li> </ul>

expression



- Use a visual T-chart to answer questions about a unit rate and ratio (for example: A T-shirt launcher can launch 5 shirts in 20 minutes. What is the rate in shirts per hour?)

T-shirts	Minutes
5	20
10	40
15	60

- Use tools or models to calculate the unit rate
- Use graphic organizers to support students' understanding of ratio and unit rate
- Understand the following concepts, symbols, and vocabulary: ratio, rate, unit rate, per, each, numerator, denominator, calculate, different, equivalent, multiple, factor

MA.6.AR.3.3

Extend previous understanding of fractions and numerical patterns to generate or complete a two- or three-column table to display equivalent part-to-part ratios and part-to-part-to-whole ratios.

**Access Point**

MA.6.AR.3.AP.3 Given a visual representation, write or select a ratio which describes the ratio relationship between part-to-part and part-to-whole ratios.

**Essential Understandings:**

- Given two groups of manipulatives, students can identify the pattern by matching the manipulatives to the picture representation
- Given two groups of manipulatives, students can identify the quantities in the relationship
- Identify the structure of a ratio in a given context (e.g., in words, with a colon, in a fraction notation)
- Demonstrate an understanding that a ratio is a comparison of two quantities
- Understand that a ratio is either part-to-whole

	<p>(some to all) or part-to-part (which must be listed in the correct order of the context)</p> <ul style="list-style-type: none"> <li>• Match/identify a simple ratio (1: X) to the relationship between two quantities</li> <li>• Given a situation, use objects or calculate to set up a ratio</li> <li>• Recognize the meaning of the placement of numbers in a ratio for a given situation</li> <li>• Write or select a ratio in three ways: number to number (1 to 2) expressed as a fraction (1/2) or using a colon (1:2)</li> <li>• Represent a part-to-whole ratio as the ratio of objects (e.g., red hats) to the total number of objects (red and green hats)</li> <li>• Represent a part-to-part ratio as the ratio of the number of one object (red hats) to the number of other objects (green hats) from a set of objects (red and green hats)</li> <li>• Understand the following concepts, symbols, and vocabulary: ratio, part-to-part, part-to-whole, rate, proportion, portions per person, portions per total. :, / , to</li> </ul>
MA.6.AR.3.4	<p>Apply ratio relationships to solve mathematical and real-world problems involving percentages using the relationship between two quantities.</p> <p><b>Access Point</b></p> <p>MA.6.AR.3.AP.4 Calculate a percentage of quantity as rate per 100 using models (e.g., percent bars or 10 x 10 grids).</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• State a relationship to a quantity out of 100</li> <li>• These will need to be very small concrete numbers (e.g., select three from an object bundle of 100)</li> <li>• Use tools to create visual representations of percentages and rates per 100</li> <li>• Use tools to convert fractions into decimals that can be displayed on a 10x10 grid</li> <li>• Use tools to convert decimals into percentages</li> <li>• Use anchor charts to support students in</li> </ul>

	<p>remembering the steps for the process of converting between forms</p> <ul style="list-style-type: none"> <li>• Understand that a fraction is expressed as a percentage by converting it to an equivalent fraction with a denominator of 100</li> <li>• Express a percentage as a fraction (<math>a/100</math>)</li> <li>• Understand that hundreds (base ten fractions) and percentages are the same, though the symbolic notation is different</li> <li>• Understand the following concepts, symbols, and vocabulary: ratio, rate, equivalent, percent, percentage, decimal, fraction, %, /</li> </ul>
MA.6.AR.3.5	<p>Solve mathematical and real-world problems involving ratios, rates and unit rates, including comparisons, mixtures, ratios of lengths and conversions within the same measurement system.</p>
	<p><b>Access Point</b></p> <p>MA.6.AR.3.AP.5a Use tools, models or manipulatives to solve problems involving ratio relationships including mixtures and ratios of length.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a scenario, use manipulatives to represent the ratio relationship       <ul style="list-style-type: none"> <li>○ (e.g., One pitcher of lemonade needs 2 cups of sugar. How many cups of sugar do you need for 2 pitchers of lemonade? OR Last year Ben’s plant was 4 inches tall, and Jenny’s was 3 inches tall. If the height of both children’s plants doubled last year, what would the ratio of the present height of Ben’s plant to the present height of Jenny’s plant?)</li> </ul> </li> <li>• Given a scenario, write or select a ratio relationship</li> <li>• Use tables, tape diagrams, or number lines to model real-life data</li> <li>• Interpret data presented in tables, tape diagrams, number lines, and manipulatives</li> <li>• Understand the following concepts, symbols, and vocabulary: data, tables, tape diagram, ratio, relationship, mixture, length, percent,</li> </ul>



	rate, symbols (:, /, to)
	MA.6.AR.3.AP.5b Use tools, models or manipulatives to solve ratio, rate or unit rate problems involving conversions within the same measurement system.
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a scenario, use manipulatives to represent the ratio relationship</li> <li>• Given a scenario, write or select a ratio relationship</li> <li>• Use tables, tape diagrams, or number lines to model real-life data</li> <li>• Interpret data presented in tables, tape diagrams, number lines, and manipulatives</li> <li>• Use Anchor charts to support students with conversion processes</li> <li>• Multiply and divide using concrete objects</li> <li>• Use a ratio to solve a measurement conversion problem</li> <li>• Multiply and divide whole numbers</li> <li>• Use the conversions on the reference sheet to solve problems within the same measurement system</li> <li>• Understand the following concepts, symbols, and vocabulary: data, tables, tape diagram, ratio, relationship, measurement terms, percent, unit rate, symbols (:, /, to)</li> </ul>

**Geometric Reasoning**

<b><i>MA.6.GR.1 Apply previous understanding of the coordinate plane to solve problems.</i></b>	
MA.6.GR.1.1	Extend previous understanding of the coordinate plane to plot rational number ordered pairs in all four quadrants and on both axes. Identify the $x$ - or $y$ -axis as the line of reflection when two ordered pairs have an opposite $x$ - or $y$ - coordinate.
	<p><b>Access Point</b></p> <p>MA.6.GR.1.AP.1 Plot integer ordered pairs in all four quadrants and on both axes.</p>

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize the axes and coordinates of labeled points on a coordinate plane</li> <li>• Identify the quadrants on a coordinate grid</li> <li>• Use tools or manipulatives to graph ordered pairs on a coordinate plane (i.e., pegboards, floor coordinate grids, straws on graph paper, smartboard)</li> <li>• Identify that in an ordered pair, the first coordinate is the location on the x-axis and the second is the location on the y-axis</li> <li>• Draw a coordinate plane and label it with the x- and y-axis</li> <li>• Locate axes where positive and negative points are found (e.g., negative numbers are found on the left and/or bottom axes)</li> <li>• Label the numbers from -10 to 10 on a number line.</li> <li>• Use coordinates to graph points on a coordinate plane</li> <li>• Use coordinates to identify points that have been plotted on a coordinate plane</li> <li>• Understand the following concepts, symbols, and vocabulary: x-axis, y-axis, quadrant, coordinate plane, coordinate, graph, order pairs, positive numbers, negative numbers, and origin</li> </ul>
MA.6.GR.1.2	<p>Find distances between ordered pairs, limited to the same <math>x</math>-coordinate or the same <math>y</math>-coordinate, represented on the coordinate plane.</p> <p><b>Access Point</b></p> <p>MA.6.GR.1.AP.2 Count the distance between two ordered pairs with the same <math>x</math> coordinate or the same <math>y</math> coordinate.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Circle two numbers on a number line and move finger when counting to find the distance between the two numbers</li> <li>• Write a subtraction sentence to find the difference between two points on a number line</li> <li>• Understand the following concepts, symbols, and vocabulary: x-axis, y-axis, coordinate plane, coordinate, graph, order pairs, positive numbers, and negative numbers</li> </ul>

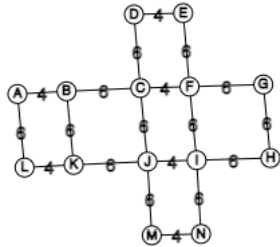
<p>MA.6.GR.1.3</p>	<p>Solve mathematical and real-world problems by plotting points on a coordinate plane, including finding the perimeter or area of a rectangle.</p> <p><b>Access Point</b></p> <p>MA.6.GR.1.AP.3 Given a rectangle plotted on the coordinate plane, find the perimeter or area of the rectangle.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Using a coordinate grid, count the length of the side of a rectangle</li> <li>• Using a coordinate grid, count the number of squares inside the rectangle to determine the area</li> <li>• Using a coordinate grid, count the distance around the outside of the rectangle to determine the perimeter</li> <li>• Using the coordinates of a figure on a coordinate grid, subtract the x or y value that changes to find the length of the side</li> <li>• Use tools to calculate the area of a rectangle using the formula <math>A=L \times W</math></li> <li>• Use tools to calculate the perimeter of a rectangle using the formula <math>P=2(L + W)</math></li> <li>• Understand the following concepts, symbols, and vocabulary: length, width, side length, distance, side, parallel, perpendicular, area, perimeter, rectangle, coordinate plane, coordinate grid math symbols: +, -, x, =</li> </ul>
<p><b><i>MA.6.GR.2 Model and solve problems involving two-dimensional figures and three- dimensional figures.</i></b></p>	
<p>MA.6.GR.2.1</p>	<p>Derive a formula for the area of a right triangle using a rectangle. Apply a formula to find the area of a triangle.</p> <p><b>Access Point</b></p> <p>MA.6.GR.2.AP.1 Given the formula, find the area of a triangle.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Identify the parts of a triangle</li> <li>• Match the parts of the triangle to the parts of the formula</li> <li>• Identify the side lengths of a triangle</li> </ul>

	<ul style="list-style-type: none"> <li>• Use tools to support substitution of side measurements into the formula</li> <li>• Use tools to calculate the area of a triangle.</li> <li>• Use formula to find the area <math>A = \frac{1}{2} (\text{Base} \times \text{Height})</math></li> <li>• Understand the following concepts, symbols, and vocabulary: base, height, area, and triangle</li> </ul>
MA.6.GR.2.2	<p>Solve mathematical and real-world problems involving the area of quadrilaterals and composite figures by decomposing them into triangles or rectangles.</p> <p><b>Access Point</b>  MA.6.GR.2.AP.2 Decompose quadrilaterals and composite figures into simple shapes (rectangles or triangles) to measure area.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Recognize simple shapes within a larger shape</li> <li>• Identify the dimensions (base, height, length, width, etc.) of smaller shapes</li> <li>• Multiply fractions and whole numbers</li> <li>• Use manipulatives, like tangrams, to support breaking composite shapes into smaller shapes</li> <li>• Use tools or manipulatives to support calculating area</li> <li>• Given a picture, identify the dimensions of two-dimensional shapes</li> <li>• Understand the following concepts, symbols, and vocabulary: quadrilaterals, rectangles, squares, triangles, area, base, height, length, width</li> </ul>
MA.6.GR.2.3	<p>Solve mathematical and real-world problems involving the volume of right rectangular prisms with positive rational number edge lengths using a visual model and a formula.</p> <p><b>Access Point</b>  MA.6.GR.2.AP.3 Given a real-world problem, find the volume of a rectangular prism using a visual model and the formula.</p> <p><b>Essential Understandings:</b></p>

	<ul style="list-style-type: none"> <li>• Recognize simple shapes within a larger shape</li> <li>• Identify the dimensions (base, height, length, width, etc.) of smaller shapes</li> <li>• Multiply fractions and whole numbers</li> <li>• Given a picture, identify the dimensions of two-dimensional and three-dimensional shapes</li> <li>• Use manipulatives, like tangrams, to support breaking composite shapes into smaller shapes</li> <li>• Use tools to support substitution of side measurements into the formula</li> <li>• Identify the faces and the base of a rectangular prism</li> <li>• Understand the following concepts, symbols, and vocabulary: polygon, rectangles, squares, volume, and prism</li> </ul>
<p>MA.6.GR.2.4</p>	<p>Given a mathematical or real-world context, find the surface area of right rectangular prisms and right rectangular pyramids using the figure’s net.</p> <p><b>Access Point</b></p> <p>MA.6.GR.2.AP.4 Find the surface area of right rectangular prisms by adding the areas of the shapes forming the two-dimensional nets.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use manipulatives (hands-on shapes) to construct and deconstruct three-dimensional figures using nets (net: unfolded form of a 3-D figure)</li> </ul> <p>e.g., cut a cereal box along its edges to form a net and allow students to take apart and reconstruct the box to see the connection</p> <ul style="list-style-type: none"> <li>• Use a picture and the vocabulary to match the three-dimensional shape to its net</li> <li>• Match a side of the net to its corresponding side on the three-dimensional shape</li> <li>• Demonstrate surface area of a cube by covering all sides</li> <li>• Demonstrate surface area of a rectangular</li> </ul>

prism by covering all sides

- Find the area of all the sides of a three-dimensional figure and add them together to find the surface area



- Understand the following concepts, symbols, and vocabulary: surface area, net, rectangular prism.

### Data Analysis and Probability

***MA.6.DP.1 Develop an understanding of statistics and determine measures of center and measures of variability. Summarize statistical distributions graphically and numerically.***

MA.6.DP.1.1

Recognize and formulate a statistical question that would generate numerical data.

**Access Point**

MA.6.DP.1.AP.1 Identify statistical questions from a list that would generate numerical data.

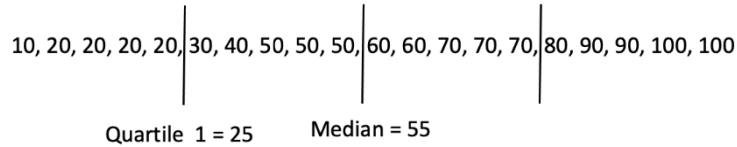
**Essential Understandings:**

- Understand the difference between statistical and nonstatistical question
- (Example) How many minutes did you work on homework last night? vs. (Non-example) Did you work on homework last night?
- Generate statistical questions
- Given a list of questions, determine which questions could have a range of answers (i.e., How old are the students in the class? How many brothers and/or sisters does each student have? etc.)
- Sort questions as statistical and nonstatistical
- Understand the following concepts, symbols,

	and vocabulary: data, statistical, nonstatistical
MA.6.DP.1.2	Given a numerical data set within a real-world context, find and interpret mean, median, mode and range.
	<p><b>Access Point</b></p> <p>MA.6.DP.1.AP.2a Use tools to identify and calculate the mean, median, mode and range represent in a set of data with no more than five elements.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use a number line to record responses in numerical order.</li> <li>• Identify the smallest number and the largest number in the range (range)</li> <li>• Create a number sentence that represents the range of responses (range)</li> <li>• Count the number of responses in each category set (mode)</li> <li>• Identify the category with the most responses (mode)</li> <li>• Use manipulatives to add the numbers in a given data set (mean)</li> <li>• Use manipulatives to divide the sum of a data set (mean)</li> <li>• Add and divide numbers in a data set using tools, as needed, to determine the mean (mean)</li> <li>• Identify the mean of a data set from manipulatives or pictorial representations (mean)</li> <li>• Identify the lowest to highest value in a data set given a number line (median)</li> <li>• Arrange data from lowest to highest (median)</li> <li>• Identify the median</li> <li>• Understand the following concepts, symbols, and vocabulary: data set, mode, most, mean, average, range, median, middle</li> </ul>
	<p>MA.6.DP.1.AP.2b Identify and explain what the mean and mode represent in a set of data with no more than five elements.</p>
	<p><b>Essential Understandings:</b></p>

	<ul style="list-style-type: none"> <li>• Use a number line to record responses in numerical order</li> <li>• Count the number of responses in each category set (mode)</li> <li>• Identify the category with the most responses (mode)</li> <li>• Identify the number/category that occurs most often in a visual display (mode)</li> <li>• Explain the mode in the context of the problem (mode)</li> <li>• Use manipulatives to add the numbers in a given data set (mean)</li> <li>• Use manipulatives to divide the sum of a data set (mean)</li> <li>• Identify the mean of a data set from manipulatives or pictorial representations (mean)</li> <li>• Add and divide numbers in a data set using tools, as needed, to determine the mean (mean)</li> <li>• Explain the mean in the context of the problem (mean)</li> <li>• Understand the following concepts, symbols, and vocabulary: data set, mode, most, mean, average.</li> </ul>
MA.6.DP.1.3	<p>Given a box plot within a real-world context, determine the minimum, the lower quartile, the median, the upper quartile and the maximum. Use this summary of the data to describe the spread and distribution of the data.</p> <p><b>Access Point</b></p> <p>MA.6.DP.1.AP.3 Given a box plot identify the value of the minimum, the lower quartile, the median, the upper quartile and the maximum.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Match the vocabulary to the corresponding part of the box plot</li> <li>• Use a number line to match the appropriate value to its corresponding parts in the box plot</li> <li>• Identify the lowest to highest value in a data set given a number line and matching symbols</li> <li>• Arrange data from lowest to highest</li> <li>• Identify the median</li> <li>• Find the lower quartile by identifying the middle value between the minimum and the median of the data set</li> <li>• Find the upper quartile by identifying the middle value between the maximum and the median of the data set</li> </ul>





- Understand the following concepts, symbols, and vocabulary: box plot, minimum, lower quartile, median, upper quartile, maximum

MA.6.DP.1.4

Given a histogram or line plot within a real-world context, qualitatively describe and interpret the spread and distribution of the data, including any symmetry, skewness, gaps, clusters, outliers and the range.

**Access Point**

MA.6.DP.1.AP.4 Given a histogram or a line plot, describe the physical features of the graph.

**Essential Understandings:**

- Use manipulative to display the frequency of a data set on a line
- Identify places on the graph where there is an increase/decrease from one point to the next
- Identify places on the graph where the data is constant from one point to the next
- Define math terms histogram, line plot, data points, increase, decrease, initial value, x and y access, range
- Use the graphing story below to help students understand that the graph models the increases and decreases of the ball bounce

MA.6.DP.1.5

Create box plots and histograms to represent sets of numerical data within real- world contexts.

**Access Point**

MA.6.DP.1.AP.5 Create histograms to represent sets of numerical data with 10 or fewer elements.

**Essential Understandings:**

- Use manipulatives to sort items by characteristics
- Sort manipulatives by characteristics and graph the quantities on a histogram
- Use the maximum and minimum values to create a range for each category
- Use anchor charts to support students in remembering the steps to create a histogram

Example:

1. Calculate the range of a data set.
2. Divide the range by the number of groups you want and round up.
3. Use the class width to create your groups.

Classes	Frequency
12 - 21	<input type="text"/>
21 - 30	<input type="text"/>
30 - 39	<input type="text"/>
39 - 48	<input type="text"/>
48 - 57	<input type="text"/>
57 - 66	<input type="text"/>

4. Find the frequency for each group.
5. Graph the frequency of each group on the histogram.

MA.6.DP.1.6

Given a real-world scenario, determine and describe how changes in data values impact measures of center and variation.

**Access Point**

MA.6.DP.1.AP.6 Calculate and identify changes (increase or decrease) in the median, mode or range when a data value is added or subtracted from a data set.

**Essential Understandings:**

- Identify the median
- Identify the mode
- Identify the range
- Determine if the added value is the same as an existing value
- Determine if the subtracted value is the same as a remaining value
- Determine if the data point in question changes the mode
- Identify the maximum and minimum of the data set
- Identify if the data point in question is less than the minimum or greater than the maximum
- Determine if the data point in question changes the range
- Determine if the data point in question changes the median

**Grade 7  
Number Sense and Operations**

***MA.7.NSO.1 Rewrite numbers in equivalent forms.***

MA.7.NSO.1.1 Know and apply the Laws of Exponents to evaluate

	<p>numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.</p> <p><b>Access Point</b></p> <p>MA.7.NSO.1.AP.1 Use properties of whole number exponents to produce equivalent expressions.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use manipulatives to demonstrate what an exponent represents (e.g., <math>8^3 = 8 \times 8 \times 8</math>).</li> <li>• Produce the correct amount of base numbers to be multiplied given a graphic organizer or template</li> <li>• Select the correct expanded form of what an exponent represents (e.g., <math>8^3 = 8 \times 8 \times 8</math>)</li> <li>• Identify the number of times the base number will be multiplied based on the exponent</li> </ul>
MA.7.NSO.1.2	<p>Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.</p> <p><b>Access Point</b></p> <p>MA.7.NSO.1.AP.2 Rewrite positive rational numbers in different but equivalent forms such as fractions, mixed numbers, repeating decimals and/or percentages to solve problems.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Demonstrate operations using manipulative when presented with common language (altogether, left over, sum, etc.)</li> <li>• Create an array of objects into groups to model the role of equal groups in a multiplication or division situation.</li> <li>• Use tools (i.e., number line, fraction tiles, calculator, and graphic representation) to model equivalent forms of numbers</li> <li>• Understand the following symbols for +, -, ×, ÷</li> <li>• Use tools, as needed, to complete the four operations</li> <li>• Solve problems using supports (Picture, Models, Representation cards, Number sentences, Mathematical word problems)</li> <li>• Match multiple representations of equivalent</li> </ul>

	<p>quantities (i.e., <math>4/2 = 2 = 2.0</math>)</p> <ul style="list-style-type: none"> <li>• Demonstrate understanding of the terms equivalent, fractions, mixed numbers, repeating decimals and percentages</li> </ul>
<b>MA.7.NSO.2 Add, subtract, multiply and divide rational numbers.</b>	
MA.7.NSO.2.1	<p>Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.</p> <p><b>Access Point</b></p> <p>MA.7.NSO.2.AP.1 Solve mathematical problems, using no more than 4 operations, with rational numbers including grouping symbols, whole-number exponents, and absolute value.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use manipulatives to represent a situation (i.e., John has five apples, and he gives some to Jim = <math>5 - x</math>)</li> <li>• When given a verbal expression (i.e., eight plus <math>y</math>), students must select the appropriate algebraic expression (<math>8 + y</math>)</li> <li>• Use tools (i.e., template, anchor chart) to support students in performing operations in the appropriate order and with numbers in different forms</li> <li>• Use tools (i.e., number line, fraction tiles, calculator, and graphic representation) to model equivalent forms of numbers</li> </ul>
MA.7.NSO.2.2	<p>Add, subtract, multiply and divide rational numbers with procedural fluency.</p> <p><b>Access Point</b></p> <p>MA.7.NSO.2.AP.2 Using tools or models, add, subtract, multiply and divide rational numbers.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Combine (+) or decompose (-) with concrete objects; use counting to get the answers</li> <li>• Combine (<math>\times</math>) or decompose (<math>\div</math>) with concrete objects; use counting to get the answers</li> <li>• Understand the symbols +, -, <math>\div</math>, =, <math>\times</math></li> <li>• Create a pictorial array for the mathematical equation and match the answer symbol (+ or -) following multiplication rules for an equation</li> <li>• Create a pictorial array for the mathematical equation and match the answer symbol (+ or -) following division rules for an equation</li> </ul>

	<ul style="list-style-type: none"> <li>• Use tools (i.e., template, anchor chart) to support students in performing operations in the appropriate order and with numbers in different forms</li> </ul>
MA.7.NSO.2.3	<p>Solve real-world problems involving any of the four operations with rational numbers.</p> <p><b>Access Point</b></p> <p>MA.7.NSO.2.AP.3 Using tools or models, solve real-world problems involving any of the four operations with rational numbers.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away, difference) in a word problem</li> <li>• Identify the purpose to find a total (sum for addition or product for multiplication), remaining amount (difference for subtraction), or one component (number of sets or number within each set-dividend or divisor for division), depending upon the words in the problem</li> <li>• Translate wording into numeric equation</li> <li>• Draw or use a representation of a word problem</li> <li>• Create a pictorial array for the mathematical equation and match the answer symbol (+ or -) following multiplication rules for an equation</li> <li>• Create a pictorial array for the mathematical equation and match the answer symbol (+ or -) following division rules for an equation</li> <li>• Combine (+) or decompose (-) with concrete objects; use counting to get the answers</li> <li>• Combine (x) or decompose (<math>\div</math>) with concrete objects; use counting to get the answers</li> <li>• Understand the symbols +, -, <math>\div</math>, =, <math>\times</math></li> <li>• Use tools (i.e., template, anchor chart) to support students in performing operations in the appropriate order and with numbers in different forms</li> </ul>

### Algebraic Reasoning

<b>MA.7.AR.1 Rewrite algebraic expressions in equivalent forms.</b>	
MA.7.AR.1.1	Apply properties of operations to add and subtract linear expressions with rational coefficients.
	<p><b>Access Point</b></p> <p>MA.7.AR.1.AP.1 Add and subtract linear expressions that include like terms.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use manipulatives to combine like terms (i.e., demonstrate <math>5x + 3x</math> by combining like manipulatives)</li> <li>• Create an array of objects for the mathematical equation and match the answer symbol (+ or -) following addition rules for an equation</li> <li>• Create an array of objects for the mathematical equation and match the answer symbol (+ or -) following subtraction rules for an equation</li> <li>• Understand the following concepts, symbols, and vocabulary for: like terms, combine, variables, positive integer, and negative integer</li> <li>• Use tools, as needed, to complete the four operations when adding and subtracting like terms</li> </ul>
MA.7.AR.1.2	Determine whether two linear expressions are equivalent.
	<p><b>Access Point</b></p> <p>MA.7.AR.1.AP.2 Use tools or manipulatives to compare two linear expressions with no more than two operations to determine whether they are equivalent.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Create an array of objects into groups to model the role of equal groups in a multiplication situation</li> <li>• Create an array of objects (e.g., two colored counters to represent positive and negative numbers) for the mathematical expression following division rules for an expression</li> <li>• Create an array of objects (e.g., two colored counters to represent positive and negative numbers) for the mathematical expression following multiplication rules for an expression</li> <li>• Use base ten blocks to represent the numbers in the distributive property (e.g., <math>4(9 + 2)</math> would</li> </ul>

look like nine blocks plus two blocks repeated four times)

- Draw a picture representing the distributive property
- Use manipulatives to combine like terms (i.e., demonstrate  $5x + 3x$  by combining 5 blue blocks and 3 blue blocks)
- Use objects to determine if the linear expressions are equal
- Use visual representations to determine if the linear expressions are equal
- Use objects to compare the linear expressions
- Use visual representations to compare the linear expressions

**MA.7.AR.2 Write and solve equations and inequalities in one variable.**

MA.7.AR.2.1

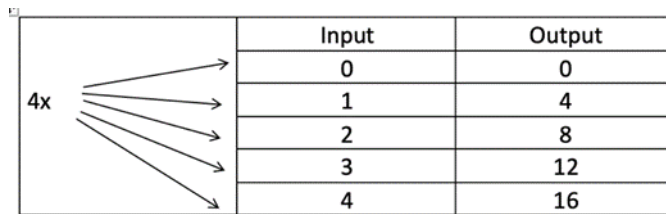
Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.

**Access Point**

MA.7.AR.2.AP.1 Select an inequality from a list that represents a real-world situation and use substitution to solve.

**Essential Understandings:**

- Use objects to compare whole numbers
- Use objects to represent inequalities with whole numbers
- Understand the following symbols  $+$ ,  $-$ ,  $\div$ ,  $=$ ,  $\times$ ,  $<$ ,  $>$ ,  $\neq$ ,  $\leq$ ,  $\geq$
- Use pictures/tables to represent inequalities with whole numbers (i.e., input/output chart or graphic organizer)



Input	Output
0	0
1	4
2	8
3	12
4	16

- Understand the following concept and vocabulary of inequality
- Evaluate an expression using substitution (e.g.,

	<p>find the value of <math>x + 4</math> when <math>x = 2</math> using manipulatives)</p> <ul style="list-style-type: none"> <li>• Use objects to solve inequalities with whole numbers</li> </ul>
<p>MA.7.AR.2.2</p>	<p>Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.</p> <p><b>Access Point</b></p> <p>MA.7.AR.2.AP.2a Set up two-step equations in one variable based on real-world problems.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Demonstrate operations using manipulative when presented with common language (altogether, left over, sum, etc.)</li> <li>• Create an array of objects into groups to model the role of equal groups in a multiplication or division situation</li> <li>• Given a set number of manipulatives, distribute them evenly to create a deficit (e.g., given 10 markers distribute 1 each to 15 students)</li> <li>• Given a set number of manipulatives, distribute them evenly to create a fraction (e.g., given 10 pieces of chalk distribute <math>\frac{1}{2}</math> piece to 20 students)</li> <li>• Use tools (i.e., template, anchor chart) to support students in performing operations to combine like terms (if needed) in the appropriate order</li> <li>• Use manipulatives to represent quantities in an equation in the form <math>px + q = r</math> from a word problem using a graphic organizer</li> <li>• Use visual cues (text marking) to support setting up the equation</li> <li>• Understand the following concepts, vocabulary, and symbols: <math>+</math>, <math>-</math>, <math>\times</math>, <math>\div</math>, <math>=</math>, <math>\neq</math>, <math>&lt;</math>, <math>&gt;</math>, equation, equal, variable, substitution</li> </ul> <p>MA.7.AR.2.AP.2b Solve two-step equations in one variable based on real-world problems, where all the terms have positive integer coefficients.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Demonstrate operations using manipulative when presented with common language (altogether, left over, sum, etc.)</li> <li>• Create an array of objects into groups to model</li> </ul>



the role of equal groups in a multiplication or division situation

- Use tools (i.e., template, anchor chart) to support students in performing operations to combine like terms (if needed) in the appropriate order
- Use manipulatives to solve real-world problems in the format  $px + q = r$ 
  - e.g., Michael paid \$15 for lunch. He paid \$3 for fries and purchased 2 burgers. How much did he pay for each burger?  $2x + 3 = 15$ 
    - Have \$15 (play money)
    - Subtract \$3 for the fries
    - Divide the remaining money between 2 groups for each burger
- Use a graphic organizer to represent quantities in an equation in the form  $px + q = r$  from a word problem
- Use anchor charts to support student in remembering the steps for solving equations and aligning key terms to operations
- Use visual cues (text marking) to support setting up and solving equations
- Identify when quantities are equal or unequal in a real-world situation. (Check your work on the problem)
- Create a pictorial array of a simple equation to translate wording
- Understand the following concepts, vocabulary, and symbols: +, -, ×, ÷, =, ≠, <, >, equation, equal, variable, substitution

***MA.7.AR.3 Use percentages and proportional reasoning to solve problems.***

MA.7.AR.3.1

Apply previous understanding of percentages and ratios to solve multi-step real- world percent problems.

**Access Point**

MA.7.AR.3.AP.1 Solve simple percentage problems in real-world contexts.

**Essential Understandings:**

- Use graphic organizers to support the set up and solving of percentage problems
- Use tools (calculator, manipulatives, and base ten

blocks) to multiply decimals

- Convert percentages to decimals; convert decimals to percentages
- Understand the relationship between fractions and percentages (e.g., 100% is one whole or 1, 50% is a one half or  $1/2$ ).
- Understand the relationship between decimals, fractions, percentages, ratios, and proportions (e.g., 100% is a whole or 1, 50% is a half or  $1/2$ ).
- Identify quantities in a problem that relate to the solution (e.g., Jane bought 3 blouses for \$5 each. Her items were on sale for 50% off. How much did Jane pay?).
- Understand the following concepts and vocabulary: percentage, decimal, fraction.

MA.7.AR.3.2

Apply previous understanding of ratios to solve real-world problems involving proportions.

**Access Point**

MA.7.AR.3.AP.2 Solve word problems involving ratios.

**Essential Understandings:**

- Locate relevant information within a word problem.
- Given a scenario, find the two quantities in a ratio (e.g., Bill has traveled 460 miles on 10 gallons of gas. Miles and gallons of gas).
- Relate the placement of numbers in a ratio to the given context (the meaning of 460:10, 460 equals miles, 10 equals a gallon of gas).
- Use a table with visuals or objects to represent a proportional relationship to solve a ratio problem.

Legs of a chair	People
4	1
8	2
12	3

- Understand the following concepts and

	vocabulary: ratio, proportion, and rate.
MA.7.AR.3.3	Solve mathematical and real-world problems involving the conversion of units across different measurement systems.
	<b>Access Point</b> MA.7.AR.3.AP.3 Use tools to solve real-world problems involving conversion of units in the same measurement system.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Multiply using concrete objects</li> <li>• Divide using concrete objects.</li> <li>• Use a ratio or ratio tables to solve a measurement conversion problem.</li> <li>• Use a pictorial representation of a ratio to solve problem.</li> <li>• Use tools to compute conversions between measurements.</li> </ul>
<b><i>MA.7.AR.4 Analyze and represent two-variable proportional relationships.</i></b>	
MA.7.AR.4.1	Determine whether two quantities have a proportional relationship by examining a table, graph or written description.
	<b>Access Point</b> MA.7.AR.4.AP.1 Given a table or a graph determine whether two quantities have a proportional relationship.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand that the relationship must be able to generate the point (0,0) in order to be a proportional relationship, even if (0,0) is not in the table.</li> <li>• Recognize input and output values in a table.</li> <li>• Use the input-output rule to generate values in a table.</li> <li>• Understand that proportional relationships intersect with the origin (0,0) on a graph.</li> <li>• Recognize a line on a graph.</li> <li>• Read a line graph with manipulatives to recognize a line on a graph.</li> <li>• Identify whether the line intersects with the origin.</li> </ul>

- Recognize the meaning of the placement of numbers in a ratio for a given situation.
- Understand the following concepts: proportional relationship, origin, input, output, coordinate, initial value, intersect, slope, constant of proportionality.

MA.7.AR.4.2

Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

**Access Point**

MA.7.AR.4.AP.2 Identify the constant of proportionality when given a table or graph of a proportional relationship.

**Essential Understandings:**

- Understand that proportional relationships intersect with the origin (0,0) on a graph.
- Recognize input and output values in a table (understand x as the input and y as the output).
- Use the input-output rule to generate values in a table.
- Identify whether the input values are increasing or decreasing.
- Identify whether the output values are increasing or decreasing.
- Given 2 points in a table, determine the change in the 2 input values.

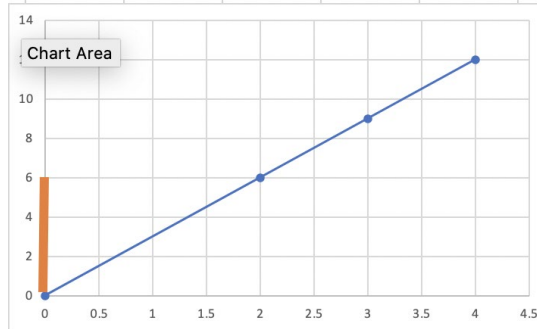
• x	• y
• 2	• 6
• 3	• 9
• 4	• 12

- Given 2 points in a table, determine the change in the 2 output values.

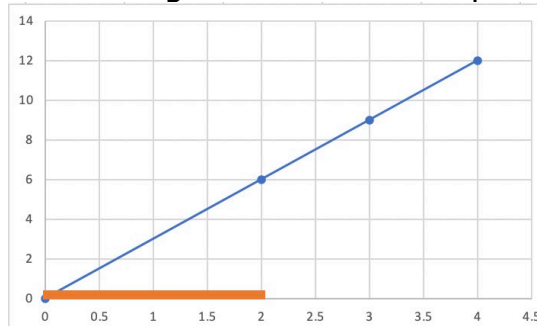
• x	• y
• 2	• 6

• 3	• 9
• 4	• 12

- Recognize that the constant of proportionality is the ratio that identifies change in output to change in input.
- Identify if the line on a coordinate plane is going up or going down to determine whether the slope is positive or negative.
- Given 2 points on the line, count the change going up and down between the 2 points.



- Given 2 points on the line, count the change going left and right between the 2 points.



- Define the term “constant of proportionality”.

MA.7.AR.4.3

Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.

**Access Point**

MA.7.AR.4.AP.3 Give a table or equation, graph a proportional relationship.

**Essential Understandings:**

- Recognize input and output values in a table (understand x as the input and y as the output).

- Given a set of input values, use substitution to complete an input-output table.
- Use the input-output rule (equation) to generate values in a table.
- Identify whether the input values are increasing or decreasing.
- Identify whether the output values are increasing or decreasing.
- Graph input/output values from a table on a coordinate plane.

MA.7.AR.4.4

Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

**Access Point**

MA.7.AR.4.AP.4 Given a table representation of a proportional relationship translate the relationship into an equation or a graph.

**Essential Understandings:**

- Recognize input and output values in a table (understand x as the input and y as the output).
- Identify the rule that creates the output value.
- Use the rule for an input-output table to create an equation.
- Identify whether the input values are increasing or decreasing.
- Identify whether the output values are increasing or decreasing.
- Graph input/output values from a table on a coordinate plane.
- Use objects to represent the quantities in the table to help identify the equation.

Legs of a chair	People	Image
4	1	

<b>8</b>	<b>2</b>	
<b>12</b>	<b>3</b>	

MA.7.AR.4.5

Solve real-world problems involving proportional relationships.

**Access Point**

MA.7.AR.4.AP.5 Solve simple real-world problems involving proportional relationships.

**Essential Understandings:**

- Relate the placement of numbers in a ratio to the given context (the meaning of 460:10, 460 equals miles, 10 equals a gallon of gas).
- Identify two equivalent ratios.
- Use tools to create equivalent ratios.
- Given a scenario, find the two quantities in a ratio (e.g., "Bill has traveled 460 miles on 10 gallons of gas. Miles and gallons of gas).
- Given a scenario, find the quantities in a proportion.
- For example: The sale price of a phone was \$150, which was only 80% of normal price. What was the normal price? The quantities are the regular price of the phone, the sale price of the phone, the percentage written as a fraction.
- Given a scenario, match the appropriate numerical value in the problem to its quantity.

$$\frac{\text{Sale price}}{\text{Original price}} = \frac{\text{Percent}}{100}$$

$$\frac{\$150}{\text{Original price}} = \frac{80}{100}$$

- Use tools (graphic organizer, table, manipulatives, etc.) to find a missing value in a proportion.
- Use tools to find the constant of variation illustrated by a proportion. For example,  $\frac{2}{3} = \frac{8}{12}$  the constant of variation is 4.
- Understand the following concepts and vocabulary: ratio, proportion, equivalent,

constant of variation, factor, fraction,  
quantity, cross multiply

## Geometric Reasoning

### ***MA.7.GR.1 Solve problems involving two-dimensional figures, including circles.***

MA.7.GR.1.1

Apply formulas to find the areas of trapezoids, parallelograms and rhombi.

**Access Point**

MA.7.GR.1.AP.1 Given the formulas, find the area of parallelograms and rhombi.

**Essential Understandings:**

- Identify the dimensions of a parallelogram or rhombi.
- Use square tiles to cover a rectangle.
- Use square tiles to cover a parallelogram or rhombus and estimate the area.
- Count the number of tiles to determine the area.
- Substitute dimensional values into the area formula.
- Use formula to find area.
- Use appropriate tools to calculate, as needed.
- Understand the following concepts and vocabulary: base, height, area, parallelogram, rhombus, and quadrilateral

MA.7.GR.1.2

Solve mathematical or real-world problems involving the area of polygons or composite figures by decomposing them into triangles or quadrilaterals.

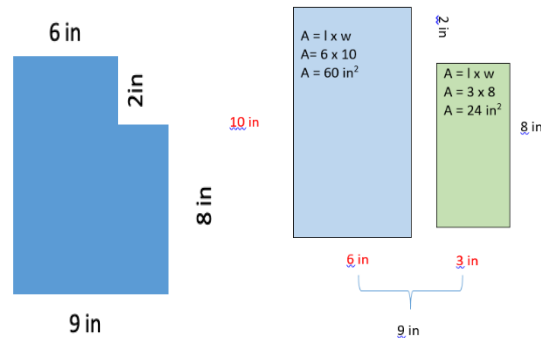
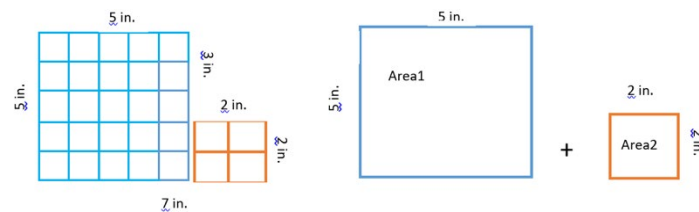
**Access Point**

MA.7.GR.1.AP.2 Decompose complex shapes (polygon, trapezoid, and pentagon) into simple shapes (rectangles, squares, triangles) to measure area.

- Recognize simple shapes within a larger shape.
- Use a grid to count dimensions in a figure.
- Identify the dimensions (base, height, length, width, etc.) of smaller shapes.



- Multiply fractions and whole numbers.
- Given a picture, identify the dimensions of two-dimensional shapes.
- Substitute dimensional values into the area formula.
- Use formula to find area.
- Use appropriate tools to calculate, as needed.
- Understand the following concepts and vocabulary: polygon, trapezoid, pentagon, rectangles, squares, triangles, area.



MA.7.GR.1.3

Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.

**Access Point**

MA.7.GR.1.AP.3 Given the formula, apply a formula for the circumference of a circle to solve mathematical problems.

**Essential Understandings:**

- Identify the radius and diameter of a circle.
- Substitute dimensional values into the circumference formula.
- Use appropriate tools to calculate, as needed.
- Understand the following concepts and vocabulary: circumference, area, pi, diameter, and radius

<p>MA.7.GR.1.4</p>	<p>Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.</p> <p><b>Access Point</b></p> <p>MA.7.GR.1.AP.4 Given the formula, apply the formula to find the area of a circle to solve mathematical problems.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Substitute dimensional values into the area formula.</li> <li>• Identify the radius and diameter of a circle.</li> <li>• Use a transparency of grid paper to place over a shape.</li> <li>• Use the grid paper to estimate the area of a circle by counting the squares and partial squares.</li> <li>• Use manipulatives, i.e., 1-inch cubes, uni-fix cubes, to estimate the area of a circle by placing the cubes on top of the circle.</li> <li>• Use appropriate tools to calculate, as needed.</li> <li>• Understand the following concepts and vocabulary: circumference, area, pi, diameter, and radius</li> </ul>
<p>MA.7.GR.1.5</p>	<p>Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.</p> <p><b>Access Point</b></p> <p>MA.7.GR.1.AP.5 Use a scale factor to draw a scale drawing of a real-world two-dimensional polygon on graph paper.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Identify when shapes are similar, but different sizes (e.g., show the same shape at various sizes – 25%, 50%, 150% –when presented with two-dimensional or three-dimensional shapes).</li> <li>• Use manipulatives to solve a one-step equation.</li> <li>• Understand that multiplying makes an object bigger and dividing makes an object smaller.</li> <li>• Use appropriate tools to solve a one-step equation.</li> <li>• Understand the following concepts and</li> </ul>

vocabulary: scale factor, polygon, two-dimensional, dimension, enlarge, reduce.

**MA.7.GR.2 Solve problems involving three-dimensional figures, including right circular cylinders.**

MA.7.GR.2.1

Given a mathematical or real-world context, find the surface area of a right circular cylinder using the figure's net.

**Access Point**

MA.7.GR.2.AP.1 Given the formula, match the parts of the formula to the right circular cylinder using the figure's net.

**Essential Understandings:**

- Identify the parts of a right circular cylinder.
- Identify the dimensions of a right circular cylinder.
- Match the parts of the figure to the parts of the net.
- Unfold three-dimensional objects into flat nets where all faces are visible.
- Recognize that surface area is found by adding up the individual areas of each face.
- Understand symbols from a formula.
- Understand the following concepts and vocabulary: area, base, height, units of measure, surface area, circle, net, face, and quadrilateral.
- Surface Area of a cylinder =  $2\pi rh + 2\pi r^2 =$  (Circumference of the circle x height) + (Area of circle 1 + Area of circle 2).

MA.7.GR.2.2

Solve real-world problems involving surface area of right circular cylinders.

**Access Point**

MA.7.GR.2.AP.2 Given the formula, use tools to find the surface area of a right circular cylinder using the figure's net.

**Essential Understandings:**

- Use appropriate tools to calculate, as needed.
- Identify the parts of a right circular cylinder.
- Identify the dimensions of a right circular cylinder.

	<ul style="list-style-type: none"> <li>• Match the parts of the figure to the parts of the net.</li> <li>• Match the parts of the net to the parts of the formula.</li> <li>• Unfold three-dimensional objects into flat nets where all faces are visible.</li> <li>• Recognize that surface area is found by adding up the individual areas of each face.</li> <li>• Understand symbols from a formula.</li> <li>• Understand the following concepts and vocabulary: area, base, height, units of measure, surface area, circle, net, face, and quadrilateral</li> <li>• Surface Area of a cylinder = <math>2\pi rh + 2\pi r^2 =</math> (Circumference of the circle x height) + (Area of circle 1 + Area of circle 2)..</li> <li>• Use square tiles to cover a figure.</li> <li>• Count the number of tiles to determine the area.</li> <li>• Calculate the area of each part of the net</li> </ul>
<p>MA.7.GR.2.3</p>	<p>Solve mathematical and real-world problems involving volume of right circular cylinders.</p> <p><b>Access Point</b></p> <p>MA.7.GR.2.AP.3 Given a formula, use tools to calculate the volume of right circular cylinders.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Identify attributes of a right circular cylinder.</li> <li>• Use base ten blocks to approximate the volume of a figure.</li> <li>• Understand two- and three-dimensional dimensionality (two-dimensional is space covered, three-dimensional is the space within).</li> <li>• Understand symbols from a formula.</li> <li>• Understand the following concepts and vocabulary: area, base, height, slant, volume, units of measure, cylinder, prism, net, and face.</li> <li>• Use appropriate tools to calculate, as needed.</li> </ul>

## Data Analysis and Probability

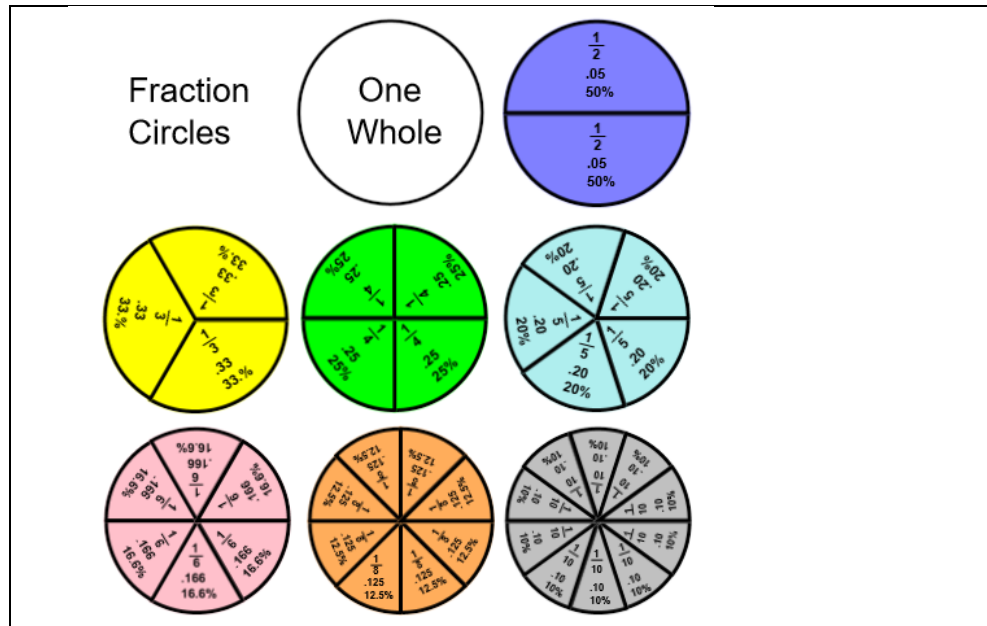
<b><i>MA.7.DP.1 Represent and interpret numerical and categorical data.</i></b>	
MA.7.DP.1.1	<p>Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers.</p>
	<p><b>Access Point</b></p> <p>MA.7.DP.1.AP.1 Use context to determine the appropriate measure of center (mean or median) or range to summarize a numerical data set with 10 or fewer elements, represented numerically or graphically.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"><li>• Understand the concept of distribution in a data set.</li><li>• Use tools to calculate the mean, median, and range in a data set.</li><li>• Identify the mean, median and range that are displayed in a bell curve, box plot, or bar graph.</li><li>• Read and interpret a display of given data (e.g., bell curve, scatter plot, box plot, or bar graph) to draw inferences (e.g., looking at the frequency of days of rain by month to determine which month I want to go camping)</li><li>• Understand the following concepts and vocabulary: ascending, descending, maximum, minimum, mean, median, range, spread of data and outlier.</li><li>• Identify the smallest number and the largest number in the range.</li><li>• Create a number sentence that represents the range of responses.</li><li>• Identify the lowest to highest value in a data set given a number line.</li><li>• Arrange data from lowest to highest.</li><li>• Use concrete materials to produce the mean (leveled plastic snap cubes).</li><li>• Find the object or manipulative in a sequence that represents the middle (median).</li><li>• Use anchor charts to support calculating the</li></ul>

	mean of a data set.
MA.7.DP.1.2	<p>Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.</p> <p><b>Access Point</b></p> <p>MA.7.DP.1.AP.2 Given two numerical or graphical representations of data in the same form, compare the mean, median, or range of each representation.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concept of distribution in a single data set and two different data sets.</li> <li>• Use tools to calculate the mean, median, and range in two different data sets.</li> <li>• Identify the mean, median and range that are displayed in a bell curve, box plot, and bar graph.</li> <li>• Use graphs or graphic organizers to compare the mean, median and range of two different data sets.</li> <li>• Identify the same measure (mean, median or range) in two different data sets.</li> <li>• Identify the lowest to highest value in a data set.</li> <li>• Arrange data from lowest to highest on a number line.</li> <li>• Identify the mean of two different data sets using manipulatives (leveled plastic snap cubes) or a line graph.</li> <li>• Use anchor charts to support calculating the mean, median and range of a data set.</li> <li>• Understand the following concepts and vocabulary: compare, ascending, descending, maximum, minimum, mean, median, range, spread of data and outlier.</li> <li>• Compare the visual of the distribution of two data sets</li> </ul>
	Given categorical data from a random sample, use proportional relationships to make predictions about a

MA.7.DP.1.3	population.
	<p><b>Access Point</b></p> <p>MA.7.DP.1.AP.3 Given data from a random sample of the population, select from a list an appropriate prediction about the population based on the data.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand basic information about a sample of a population</li> <li>• Identify a representation of two bar graphs (one category apiece) as having greater or less frequency of members/events related to a single variable (e.g., compare number of boys in soccer to girls in two graphs) <ul style="list-style-type: none"> <li>• Identify characteristics of a population and of its random sample</li> <li>• Understand that the characteristics of a random sample should be similar to its population</li> <li>• Given a population and a random sample, identify which is the random sample and which is the population</li> <li>• Select statements from a list that apply to a given sample</li> <li>• Select statements from a list that apply to a given data set</li> <li>• Identify potential inferences when given data from a sample</li> <li>• Select statements from a list that make predictions about a random sample based on the characteristics of the population</li> <li>• Understand the following concepts and vocabulary: statistics, inference, conclusion, estimation, probability (likelihood), prediction and hypothesis testing (cause/effect), more, less, higher, lower, random, characteristic, ratio, proportional, and prediction</li> <li>• Understand that statistics is collecting, organizing, analyzing, and interpreting data in order to make decisions</li> <li>• Understand that each item/subject in a random sample has the same chance of being selected</li> </ul> </li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that generalizations are only valid if they are based on similar characteristics in both the sample and the population</li> <li>• Understand that decisions about the population can be made based on the information gathered from the random sample</li> </ul>
<p>MA.7.DP.1.4</p>	<p>Use proportional reasoning to construct, display and interpret data in circle graphs.</p>
	<p><b>Access Point</b></p> <p>MA.7.DP.1.AP.4 Use proportional reasoning to interpret data in a pie chart.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Match the data category with its data</li> <li>• Identify differences in the sizes of the circle graph sections in relation to each other</li> <li>• Compare the values of the categories in the data set</li> <li>• Order categories of the data set based on their relative size or percentage</li> <li>• Identify the largest and smallest categories in the data set</li> <li>• Match the percent values to their fraction value</li> <li>• Select statements from a list that describe the data set</li> <li>• Use fraction circle manipulatives to identify fractional representations</li> <li>• Use tools to calculate the percent value of a category from the data set</li> <li>• Use tools to calculate the data quantity when given the percent value of a data set</li> <li>• Use manipulatives (pie shaped) to build a circle</li> </ul>





MA.7.DP.1.5

Given a real-world numerical or categorical data set, choose and create an appropriate graphical representation.

**Access Point**

MA.7.DP.1.AP.5 Given a data set, select an appropriate graphical representation (histogram, bar chart, or line plot).

**Essential Understandings:**

- Use manipulatives to display the frequency of a data set on a line
- Identify what a data point represents
  - Understand that each point may represent more than one item (i.e., the car on a histogram may represent five cars although only one is pictured)
  - Match a frequency table with its data plot
  - Select a data display that best fits a given set of information
  - Use manipulatives to create a simple representation of the data from a frequency table
  - Match a point on a graph as being part of a real-world data set for a given line or bar
  - Match or plot the points from a data table on a graph
  - Determine if a point is or is not on a graphical

display

**MA.7.DP.2 Develop an understanding of probability. Find and compare experimental and theoretical probabilities.**

MA.7.DP.2.1

Determine the sample space for a simple experiment.

**Access Point**

MA.7.DP.2.AP.1 Use tree diagrams, frequency tables, organized lists, and/or simulations to collect data from a simple experiment.

**Essential Understandings:**

- Use items like coins to determine the probability of an outcome (1/2 heads)
  - Using manipulatives and a chart to capture the outcomes of coin flips or dice rolls
  - Identify the formula for finding experimental probability of an event (experimental probability of an event = number of times it actually happened/total number of outcomes)
  - Use a chart to capture the outcomes of coin flips or dice rolls
  - Use a tree diagram to display the possible options for outcomes
  - Use a frequency table or organized list to record the outcomes from an experiment
  - Given a chance event, find the probability using a manipulative. For example, the probability of landing on yellow =  $\frac{1}{4}$  or 0.25



- Understand probability notation, for example  $P(\text{heads}) = \frac{1}{2}$
- Understand the concepts, symbols, and vocabulary: probability, likelihood

MA.7.DP.2.2

Given the probability of a chance event, interpret the likelihood of it occurring. Compare the probabilities of

	<p>chance events.</p> <p><b>Access Point</b></p> <p>MA.7.DP.2.AP.2 Given the probability of a simple event written as a fraction, percentage or decimal between 0 and 1, determine how likely is it that an event will occur.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the value for probability of a chance event ranges between 0 and 1</li> <li>• Understand probabilities close to 1 correspond to highly likely events and probabilities close to 0 correspond to highly unlikely events</li> <li>• Given a set of items. Identify items that are in the set and items that are not in the set</li> <li>• Identify the formula for finding experimental probability of an event <ul style="list-style-type: none"> <li>○ (Experimental probability of an event = number of times it actually happened/total number of outcomes)</li> <li>• Given a chance event, find the probability of an outcome that is likely</li> <li>• For example, the probability of rolling a 2 with a die is <math>\frac{1}{6}</math> or 0.166...</li> <li>• Given a chance event, find the probability of an outcome that is highly unlikely</li> <li>• For example, the probability of rolling a B on a number die is <math>\frac{0}{6}</math> or 0</li> <li>• Given a chance event, find the probability of an outcome that is highly likely</li> <li>• For example, the probability of pulling a marble out of a bag of 5 marbles is <math>\frac{5}{5}</math> or 1</li> <li>• Use tools to convert fractions to decimals</li> <li>• Match a scenario with its likelihood</li> <li>• Use a model to identify the likelihood of a chance event</li> <li>• Understand the following concepts, symbols, and vocabulary: chance event, probability, likelihood, outcome, event, simple event</li> </ul> </li> </ul>
MA.7.DP.2.3	<p>Find the theoretical probability of an event related to a simple experiment.</p> <p><b>Access Point</b></p>

	<p>MA.7.DP.2.AP.3 Determine the theoretical probability of a simple event.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand that sample space is the set of all possible outcomes of an experiment</li> <li>• Create/recognize possible outcomes</li> <li>• Given a set of items, identify the probability of selecting a specific item from the set</li> <li>• Use items like coins to determine the probability of an outcome (1/2 heads)</li> <li>• Identify the formula for finding the theoretical probability of an event (probability of an event happening = number of ways it can happen/total number of outcomes)</li> <li>• Understand the following concepts, symbols, and vocabulary: probability, likelihood, outcome, sample space, trial</li> </ul>
<p>MA.7.DP.2.4</p>	<p>Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.</p> <p><b>Access Point</b></p> <p>MA.7.DP.2.AP.4 Conduct a simple experiment to find experimental probabilities</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Using manipulatives and a chart to capture the outcomes of coin flips or dice rolls</li> <li>• Use technology generated outcomes to find experimental probabilities for an event (i.e., Random.org, justflipacoin.com)</li> <li>• Use tools to calculate the probability of a simple chance event <ul style="list-style-type: none"> <li>• Understand that sample space is the set of all possible outcomes (combinations) of an experiment</li> </ul> </li> <li>• Match an outcome to its theoretical probability from a list <ul style="list-style-type: none"> <li>• Match an outcome to its experimental probability from the sample space</li> <li>• Identify the possible outcomes for a specified probability in the sample space</li> <li>• Count the number of times the specified</li> </ul> </li> </ul>

	<p>outcome occurs in the sample space</p> <ul style="list-style-type: none"> <li>• Identify the formula for finding experimental probability of an event (experimental probability of an event = number of times it actually happened/total number of outcomes)</li> <li>• Given a chance event, find the probability of an outcome that is likely</li> <li>• For example, the probability of rolling a 2 with a die is <math>1/6</math> or 0.166...</li> <li>• Given a chance event, find the probability of an outcome that is highly unlikely</li> <li>• For example, the probability of rolling a B on a number die is <math>0/6</math> or 0</li> <li>• Given a chance event, find the probability of an outcome that is highly likely</li> <li>• For example, the probability of pulling a marble out of a bag of 5 marbles is <math>5/5</math> or 1</li> <li>• Use a model to identify the probability of a chance event</li> <li>• Understand the following concepts, symbols, and vocabulary: more, less, same, different, equal, compare, chance event, probability, likelihood, outcome, event, compound event, simple event</li> </ul>
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**Grade 8**

**Number Sense and Operations**

<b><i>MA.8.NSO.1 Solve problems involving rational numbers, including numbers in scientific notation, and extend the understanding of rational numbers to irrational numbers.</i></b>	
MA.8.NSO.1.1	Extend previous understanding of rational numbers to define irrational numbers within the real number system. Locate an approximate value of a numerical expression involving irrational numbers on a number line.
	<b>Access Point</b> MA.8.NSO.1.AP.1 Locate approximations of irrational numbers on a number line.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Locate whole numbers on a number line</li> </ul>

	<ul style="list-style-type: none"> <li>• Locate decimal numbers on a number line</li> <li>• Locate fractions on a number line</li> <li>• Use a calculator to find the square root of a number</li> <li>• Use the square root of a number to place a value on the number line</li> <li>• Round an irrational number to the nearest whole number</li> <li>• Round an irrational number to the nearest tenths place</li> <li>• Round an irrational number to the hundredths place</li> <li>• Round an irrational number to the thousandths place</li> <li>• Understand the following concepts, symbols, and vocabulary for: rational number, irrational number, square root, pi, fraction, decimal</li> </ul>
MA.8.NSO.1.2	<p>Plot, order and compare rational and irrational numbers, represented in various forms.</p> <p><b>Access Point</b></p> <p>MA.8.NSO.1.AP.2 Use appropriate tools to plot, order, and compare simple square roots and cube roots for quantities less than 100.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use manipulatives to make a square.</li> </ul> <p>The area of the square is the perfect square The length of each side is the square root</p> <ul style="list-style-type: none"> <li>• Use manipulatives to make a cube</li> </ul> <p>The volume of the cube is the perfect cube The length of each side is the cube root</p> <ul style="list-style-type: none"> <li>• Use a multiplication table to identify perfect squares and square roots</li> <li>• Use a calculator to determine the squares and cubes of numbers ranging from -6 to 6</li> <li>• Use a calculator to determine the square roots of numbers ranging from 0 to 36</li> <li>• Use a calculator to determine the cubed roots of numbers ranging from -100 to 100</li> </ul>

- Identify the square and cube functions on a calculator
- Identify the square root and cube root function button on a calculator

#### Plot

- Locate whole numbers on a number line
- Use the square of a number to place a value on the number line (i.e.,  $3^2 = 9$ , plot 9 on the number line)
  - Use the square root of a number to place a value on the number line (i.e., square root of  $9 = 3$ , plot 3 on the number line)
- Use the cube of a number to place a value on the number line
- Use the cube root of a number to place a value on the number line

#### Order

- Understand the relative size of quantities
- Identify quantities that are increasing or decreasing in size based on their relative value using manipulatives
- Arrange quantities using manipulatives based on their relative size
- Use a "number path" to order quantities

#### Compare

- Use graphic organizers to compare two quantities
- Use manipulatives to compare two quantities
- Identify the least and greatest quantities
- Use appropriate tools to order the values represented by the quantities
- Use inequality symbols to compare quantities using manipulatives (i.e., snap cubes, base 10 blocks, etc.)
- Use inequality symbols to compare quantities using representations of the quantities (i.e., tally marks, pictures, numbers, etc.)
- Compare the values of the digits of two different numbers starting with the greatest place value position in order to identify which

	decimal is greater and which is lesser
MA.8.NSO.1.3	Extend previous understanding of the Laws of Exponents to include integer exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to integer exponents and rational number bases, with procedural fluency.
	<p><b>Access Point</b></p> <p>MA.8.NSO.1.AP.3 Use the properties of integer exponents and product/quotient of powers with like bases to produce equivalent expressions.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Expand or simplify an expression (i.e., <math>5^3 = 5 \times 5 \times 5 = 125</math>)</li> <li>• Use the Product of Powers to simplify the expression: <ul style="list-style-type: none"> <li><b>PRODUCT OF POWERS</b></li> <li>For any number b and all integers x and y:</li> <li><math>b^x \cdot b^y = b^{x+y}</math></li> <li><math>2^3 \cdot 2^4 = 2^{3+4} = 2^7</math></li> </ul> </li> <li>• Use the Quotient of Powers to simplify the expression: <ul style="list-style-type: none"> <li><b>QUOTIENT OF POWERS</b></li> <li>For any non-zero number x and any integers a and b:</li> <li><math>\frac{x^a}{x^b} = x^{a-b}</math></li> <li><math>\frac{x^4}{x^2} = x^{4-2}</math></li> </ul> </li> <li>• Add and subtract integers (e.g., use manipulatives, a number line or calculator to add <math>2 + -5</math>) <ul style="list-style-type: none"> <li>• Identify the number of times the base number will be multiplied based on the exponent</li> <li>• Understand that a negative exponent will result in a fraction with a numerator of 1 (for example, <math>25^{-1} = 1/25</math>)</li> <li>• Use manipulatives to demonstrate what an exponent represents (e.g., <math>8^3 = 8 \times 8 \times 8</math>)</li> <li>• Select the correct expanded form of what an exponent represents (e.g., <math>8^3 = 8 \times 8 \times 8</math>)</li> <li>• Produce the correct amount of base numbers to be multiplied given a graphic organizer or</li> </ul> </li> </ul>



	<p>template</p> <ul style="list-style-type: none"> <li>• Match an expression to its exponential expansion</li> <li>• Match an expression to its exponential expansion using manipulatives or pictorial representation</li> <li>• Use manipulatives to simplify an expression</li> <li>• Use tools (i.e., graphic organizer, manipulatives, etc.) to combine like terms.</li> <li>• Understand the following concepts, symbols, and vocabulary: base number, exponent, integer, expand, like terms</li> </ul>
MA.8.NSO.1.4	<p>Express numbers in scientific notation to represent and approximate very large or very small quantities. Determine how many times larger or smaller one number is compared to a second number.</p> <p><b>Access Point</b></p> <p>MA.8.NSO.1.AP.4 Multiply a single-digit number by the power of 10 using a calculator. Identify whether the number in scientific notation represents a very large or very small quantity.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use base ten blocks to multiply a single digit number by 10, 100, 1000, etc.</li> <li>• Use a calculator to multiply a single digit number by a power of 10</li> <li>• Identify the manipulative that represents the power of 10</li> <li>• Identify the product of powers of ten</li> </ul>
MA.8.NSO.1.5	<p>Add, subtract, multiply and divide numbers expressed in scientific notation with procedural fluency.</p> <p><b>Access Point</b></p> <p>MA.8.NSO.1.AP.5 Perform operations with numbers expressed in scientific notation using a calculator.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Select the appropriate base ten bundle to represent the number expressed in scientific notation <ul style="list-style-type: none"> <li>• Match the exponential form to the standard form of a number</li> </ul> </li> <li>• Understand the following concepts, symbols,</li> </ul>

	<p>and vocabulary: scientific notation, base number/digit term, exponent, positive and negative numbers</p> <ul style="list-style-type: none"> <li>• Select the correct numeric representation for a given question (e.g., <math>5 \times 10^{-9}</math>)</li> <li>• Use a calculator to perform operations on numbers expressed in scientific notation</li> <li>• Identify the operations to be performed on the numbers expressed in the scientific notation</li> <li>• Determine the relative size of the answer based on the operation</li> <li>• Determine the sign of the answer based on the terms in the problem</li> </ul>
MA.8.NSO.1.6	<p>Solve real-world problems involving operations with numbers expressed in scientific notation.</p> <p><b>Access Point</b></p> <p>MA.8.NSO.1.AP.6 Given a real-world problem, perform operations with numbers expressed in scientific notation using a calculator and interpret the answer in context.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use properties of integer exponents to simplify expressions</li> <li>• Identify what an exponent represents</li> <li>• Solve numerical equations involving whole number bases and exponents</li> <li>• Apply the concepts of addition, subtraction, multiplication, and division <ul style="list-style-type: none"> <li>• Draw a picture or use manipulatives to understand the different parts of an expression</li> </ul> </li> <li>• Match the exponential form to the standard form of a number <ul style="list-style-type: none"> <li>• Select the correct numeric representation for a given question (e.g., <math>5 \times 10^{-9}</math>).</li> <li>• Use a calculator to perform operations on numbers expressed in scientific notation</li> <li>• Identify the operations to be performed on the numbers expressed in the scientific notation</li> <li>• Determine the relative size of the answer</li> </ul> </li> </ul>

based on the operation

- Determine the sign of the answer based on the terms in the problem
- Add and subtract integers (e.g., use manipulatives, a number line or calculator to add  $2 + -5$ )
  - Identify the number of times the base number will be multiplied based on the exponent
  - Understand that a negative exponent will result in a fraction with a numerator of 1 (for example,  $25^{-1} = 1/25$ )
  - Use manipulatives to demonstrate what an exponent represents (e.g.,  $8^3 = 8 \times 8 \times 8$ )
  - Select the correct expanded form of what an exponent represents (e.g.,  $8^3 = 8 \times 8 \times 8$ )
  - Produce the correct amount of base numbers to be multiplied given a graphic organizer or template
  - Match an expression to its exponential expansion
  - Match an expression to its exponential expansion using manipulatives or pictorial representation
  - Use manipulatives to simplify an expression
  - Use tools (i.e., graphic organizer, manipulatives, etc.) to combine like terms
- Identify the quantities in the problem
- Label the numerical terms with their quantity labels
- Use a graphic organizer to set up a numerical expression
- Use a graphic organizer to solve and interpret the answer
- Use pictorial representations to interpret the answer
- Use text marking strategies to identify the quantities and their numerical values
  - Understand the following concepts, symbols, and vocabulary: scientific notation, base number/digit term, exponent, positive and negative numbers

MA.8.NSO.1.7	Solve multi-step mathematical and real-world problems involving the order of operations with rational numbers including exponents and radicals.
	<p><b>Access Point</b></p> <p>MA.8.NSO.1.AP.7 Use tools to solve multi-step mathematical problems, with four or fewer steps, involving the order of operations with rational numbers including exponents and perfect squares and/or square roots.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Create an array of objects into groups to model the role of equal groups in a multiplication or division situation</li> <li>• Create a pictorial array for the mathematical equation and match the answer symbol (+ or -) following multiplication/division rules for an equation</li> <li>• Use tools or objects, as needed, to solve expressions using the order of operations</li> <li>• Use tools or objects to simplify exponents and perfect squares and/or square roots</li> <li>• Use tools (i.e., template, anchor chart) to support students in performing operations in the appropriate order and with numbers in different forms</li> <li>• Use tools (i.e., number line, fraction tiles, calculator, graphic representation) to model equivalent forms of numbers.</li> <li>• Evaluate an expression using substitution with manipulatives (e.g., find the value of <math>x + 4</math> when <math>x = 2</math> using manipulatives)</li> <li>• Use tools or models, like an input/output table or number line, to solve expressions using substitution</li> <li>• Understand the following concepts, symbols, and vocabulary: expression, substitution, operations, input, output, set, simplify, variable, exponents, perfect squares, square root, math symbols +, -, <math>\div</math>, <math>\times</math>, <math>\sqrt{\quad}</math></li> </ul>

**Algebraic Reasoning**

***MA.8.AR.1 Generate equivalent algebraic expressions.***

MA.8.AR.1.1	Apply the Laws of Exponents to generate equivalent algebraic expressions, limited to integer exponents and monomial bases.
	<p><b>Access Point</b></p> <p>MA.8.AR.1.AP.1 Use the properties of integer exponents and product/quotient of powers with like bases to produce equivalent algebraic expressions limited to positive exponents and monomial bases.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Apply the concepts of addition, subtraction, multiplication, and division <ul style="list-style-type: none"> <li>• Draw a picture or use manipulatives to understand the different parts of an expression</li> </ul> </li> <li>• Use tools to solve a numerical expression with whole numbers <ul style="list-style-type: none"> <li>• Locate an exponent in an expression</li> </ul> </li> <li>• Produce the correct amount of base numbers to be multiplied given a graphic organizer or template</li> <li>• Select the correct expanded form of what an exponent represents (e.g., <math>8^3 = 8 \times 8 \times 8</math>) <ul style="list-style-type: none"> <li>• Identify the number of times the base number will be multiplied based on the exponent</li> <li>• Expand or simplify an expression (i.e., <math>5^3 = 5 \times 5 \times 5 = 125</math>)</li> <li>• Use tools (i.e., template, anchor chart) to support students in performing operations in the appropriate order and with numbers in different forms</li> <li>• Use tools (i.e., number line, fraction tiles, calculator, and graphic representation) to model equivalent forms of numbers</li> </ul> </li> <li>• Use the Product of Powers to expand or simplify an expression:</li> </ul>

**PRODUCT OF POWERS**

For any number b and

all integers x and y:

$$b^x \cdot b^y = b^{x+y}$$

$$2^3 \cdot 2^4 = 2^{3+4} = 2^7$$

- Use the Quotient of Powers to expand or simplify an expression:

**QUOTIENT OF POWERS**

For any non-zero number x

and any integers a and b:

$$\frac{x^a}{x^b} = x^{a-b}$$

$$\frac{x^4}{x^2} = x^{4-2}$$

- Understand the following concepts, symbols, and vocabulary for exponent

MA.8.AR.1.2

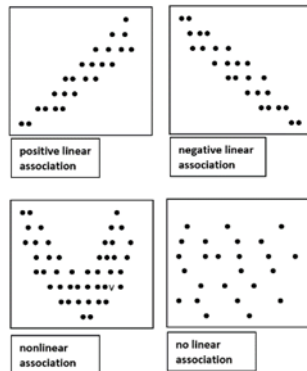
Given a scatter plot within a real-world context, describe patterns of association.

**Access Point**

MA.8.AR.1.AP.2 Use the distributive property to multiply a monomial by a linear expression.

**Essential Understandings:**

- Identify a similar distribution when given a choice of three (e.g., when shown a positive association, can select a second example of a positive association from three choices)
- Identify the associations between the variables using supports (e.g., use a template to determine the association, use a pre-made scatter plot transparency and place on top of a given scatter



- Locate points on the x-axis and y-axis on an adapted grid (not necessarily numeric)
- Describe, in general, the direction of the points on the grid (for example, the points are increasing from left to right)
- Understand the following concepts and vocabulary: best fit line, variable, outliers, linear, nonlinear, positive association, negative association, no association

MA.8.AR.1.3

Given a scatter plot with a linear association, informally fit a straight line.

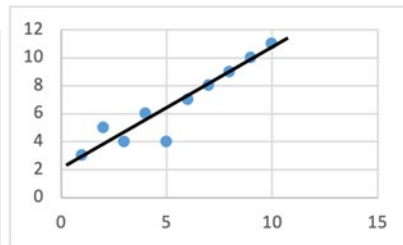
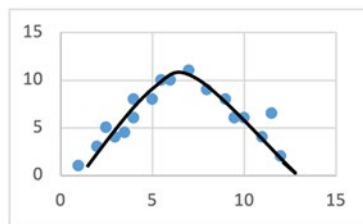
**Access Point**

MA.8.DP.1.AP.3 Given a scatter plot with a linear association, use tools to draw or place a line of best fit.

**Essential Understandings:**

- Draw a line between two points on a graph
- Draw a line between at least two points on the graph that approximates the general direction of the data
- Identify linear and non-linear associations in various scatter plots

For example:



- Identify when data points are close together or spread out (strong or weak association)

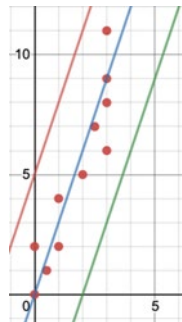
For example:

Strong Positive Linear Association	Weak Positive Linear Association
<p>A scatter plot on a grid showing a strong positive linear association. The data points are tightly clustered around a straight line that slopes upwards from left to right.</p>	<p>A scatter plot on a grid showing a weak positive linear association. The data points are widely scattered around a straight line that slopes upwards from left to right.</p>

- Use appropriate tools (uncooked spaghetti noodle, clear ruler, Popsicle stick, etc.) to visually approximate the line of best fit



- Understand the following concepts and vocabulary: best fit line, variable, outliers, scatter plots, data points, linear associations, nonlinear associations
- Given three choices, select the line of best fit



***MA.8.AR.2 Solve multi-step one-variable equations and inequalities***

MA.8.AR.2.1

Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.

**Access Point**

MA.8.AR.2.AP.1a Identify the steps to solve a given multi-step equation in one variable, with integers coefficients. Include equations with variables on both sides.

**Essential Understandings:**

- Identify the operations in an equation
- Identify the inverse operation in order to solve a single step in the equation
- Identify like terms and their opposites in the equation, when necessary



- Identify steps to simplify each side of the equation, when possible
- Order a list of steps to solve an equation

MA.8.AR.2.AP.1b Solve multi-step equations in one variable, with integers coefficients. Include equations with variables on both sides.

**Essential Understandings:**

- Use manipulatives or a graphic organizer to set up and solve a problem
  - Identify the inverse operation in order to solve one-step equations
  - Identify like terms in the equation

$$3x + 2x = 12 - x$$

- Combine like terms in the equation, when possible

$$8 + 3x = 5x - 2$$

$$8 + 2 + 3x = 5x - 2 + 2$$

$$10 + 3x = 5x$$

$$10 + 3x - 3x = 5x - 3x$$

$$10 = 2x$$

- Simplify equations by combining terms, using the properties or inverse operations
- Identify inverse operations and use them to solve equations.
- Understand the following concepts, vocabulary, and symbols: +, -, ×, ÷, =, variable, like terms, reciprocal, inverse operation, and equation

MA.8.AR.2.2

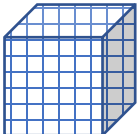
Solve two-step linear inequalities in one variable and represent solutions algebraically and graphically.

**Access Point**

MA.8.AR.2.AP.2 Select a two-step inequality from a list that represents a real-world situation and use substitution to solve.

**Essential Understandings:**

- Match an inequality to its real-world context
  - Identify the variable in the inequality
  - Create a pictorial array of an inequality to translate wording
  - Use tools, (i.e., manipulatives, algebra tiles,

	<p>software, equation calculators, etc.) to substitute into inequalities</p> <ul style="list-style-type: none"> <li>• Understand the following vocabulary and symbols: +, -, ×, ÷, =, &lt;, &gt;, ≤, ≥ linear, variable, inequality, equation, exponent, rational</li> </ul>
<p>MA.8.AR.2.3</p>	<p>Given an equation in the form of <math>xx^2 = pp</math> and <math>xx^3 = qq</math>, where <math>pp</math> is a whole number and <math>qq</math> is an integer, determine the real solutions.</p>
	<p><b>Access Point</b></p> <p>MA.8.AR.2.AP.3 Given an equation in the form of <math>x^2 = p</math> and <math>x^3 = q</math>, use tools to determine real solutions where <math>p</math> is a perfect square up to 144 and <math>q</math> is a perfect cube from -125 to 125.</p>
	<ul style="list-style-type: none"> <li>• Use manipulatives to make a square The area of the square is the perfect square The length of each side is the square root</li> <li>• Use manipulatives to make a cube</li> </ul> <p>The volume of the cube is the perfect cube</p>  <p>The length of each side is the cube root</p> <ul style="list-style-type: none"> <li>• Use a multiplication table to identify perfect squares</li> <li>• Identify the square root and cube root function button on a calculator</li> <li>• Use a calculator to determine the square roots and cube roots of numbers from -125 to 125</li> <li>• Use a calculator to find the squares and cubes of numbers ranging from -12 to 12</li> <li>• Use manipulatives to divide numbers</li> <li>• Identify the characteristics of an irrational number</li> <li>• Identify non-perfect square roots and cube roots</li> <li>• Understand the following vocabulary and symbols: square root, cube root, squaring, cubing, square root symbol, cube root symbol,</li> </ul>

	perfect square, perfect cube, exponent, factor, multiple
<b>MA.8.AR.3 Extend understanding of proportional relationships to two-variable linear equations.</b>	
MA.8.AR.3.1	Determine if a linear relationship is also a proportional relationship.
	<b>Access Point</b> MA.8.AR.3.AP.1 Given a table, a graph, or equation, determine whether a linear relationship is proportional.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Identify the origin in a table or a graph</li> <li>• Identify whether the linear relationship crosses the origin in a graph</li> <li>• In a table or equation, use substitution to determine if the origin is a solution to the linear relationship</li> <li>• Use manipulatives to solve the equation or build the graph to determine whether the linear relationship contains the origin</li> <li>• Determine whether the equation is in the form <math>y=kx</math> (proportional) or <math>y=kx + c</math> (non-proportional), where <math>k</math> and <math>c</math> are constants and not equal 0</li> <li>• Understand the following concepts, vocabulary, and symbols: linear, nonlinear, linear relationship, origin, proportional</li> </ul>
MA.8.AR.3.2	Given a table, graph or written description of a linear relationship, determine the slope.
	<b>Access Point</b> MA.8.AR.3.AP.2 Given a table or graph of a linear relationship, identify the slope.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Identify properties of a linear relationship on a graph (e.g., slope, increasing or decreasing, where does it cross the x- and y-axis)</li> <li>• Given a table, identify if a linear relationship <ul style="list-style-type: none"> <li>• Given an equation, determine the slope</li> </ul> </li> </ul> <p>For example, if <math>y=6x</math>, the slope is 6</p> <ul style="list-style-type: none"> <li>• Understand that the slope, constant of proportionality and rate of change all describe a change in a mathematical relationship</li> </ul>

	<ul style="list-style-type: none"> <li>• In a table, identify the slope as the change in y over the change in x (give an example)</li> <li>• Understand the following concept, vocabulary, and symbol: function, slope, constant of proportionality, increase, decrease, rate of change, x-axis, y-axis, coordinate, rise, and run</li> </ul>
<p>MA.8.AR.3.3</p>	<p>Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.</p> <p><b>Access Point</b></p> <p>MA.8.AR.3.AP.3 Given a table or graph of a linear relationship, identify from a list, the equation in slope-intercept form.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Identify the slope, coordinate and/or y-intercept of an equation in slope intercept form</li> <li>• Identify the change in the x variable and the change in y variable given a table</li> <li>• Identify properties of a linear relationship on a graph (e.g., slope, increasing or decreasing, where does it cross the x- and y-axis) <ul style="list-style-type: none"> <li>• Identify the slope and the y-intercept of a graph or table</li> <li>• Identify where the function increases or decreases on a graph</li> <li>• Match the graph to a given slope and y-intercept</li> <li>• Match a table to a given slope and y-intercept</li> <li>• Identify where the linear relationship increases or decreases on a graph</li> <li>• Indicate the point on a line that crosses the y-axis</li> <li>• Count the distance up/down between two points on the coordinate plane (rise)</li> <li>• Count the distance to the right, between two points on the coordinate plane (run)</li> <li>• Understand the following concepts and vocabulary: x-axis, y-axis, x-intercept, y-intercept, line, rise, run, fall, slope, rate of change</li> </ul> </li> </ul>

	<ul style="list-style-type: none"> <li>• Interpret/define a line graph with coordinates for multiple points</li> <li>• Identify coordinates (points) on a graph</li> </ul>
MA.8.AR.3.4	<p>Given a mathematical or real-world context, graph a two-variable linear equation from a written description, a table or an equation in slope-intercept form.</p> <p><b>Access Point</b></p> <p>MA.8.AR.3.AP.4 Graph a two-variable linear equation from a table or an equation in slope-intercept form.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Identify the slope of the equation in slope intercept form</li> <li>• Identify the y-intercept of the equation in slope intercept form</li> <li>• Identify and graph coordinates from a table</li> <li>• Identify the y-intercept from a table</li> <li>• Identify the slope from a table</li> <li>• Identify whether the line will increase or decrease from a table</li> <li>• Identify whether the line will increase or decrease from the equation <ul style="list-style-type: none"> <li>• Identify the slope and the y-intercept of a graph</li> <li>• Draw a sketch given a point and a y-intercept</li> <li>• Draw a sketch given a slope and a y-intercept</li> <li>• Match the graph to a given slope and y-intercept</li> <li>• Identify where the linear relationship increases or decreases on a graph</li> <li>• Indicate the point on a line that crosses the y-axis</li> <li>• Count the distance up/down between two points on the coordinate plane (rise)</li> <li>• Count the distance to the right, between two points on the coordinate plane (run)</li> <li>• Understand the following concepts and vocabulary: x-axis, y-axis, x-intercept, y-intercept, line, rise, run, fall, slope, rate of change</li> </ul> </li> <li>• Interpret/define a line graph with coordinates</li> </ul>

	<p>for multiple points</p> <ul style="list-style-type: none"> <li>• Identify coordinates (points) on a graph</li> </ul>
<p>MA.8.AR.3.5</p>	<p>Given a real-world context, determine and interpret the slope and <math>tt</math>-intercept of a two-variable linear equation from a written description, a table, a graph or an equation in slope-intercept form.</p> <p><b>Access Point</b></p> <p>MA.8.AR.3.AP.5 Given a real-world context, identify the slope and <math>y</math>-intercept of a two-variable linear equation from a table, a graph or an equation in slope-intercept form.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Identify coordinates (points) on a graph</li> <li>• Count the distance up/down between two points on the coordinate plane (rise)</li> <li>• Count the distance to the right, between two points on the coordinate plane (run)</li> <li>• Identify the slope of the equation in slope intercept form</li> <li>• Identify the slope of the equation in slope intercept form within the context of the problem</li> <li>• Identify the <math>y</math>-intercept of the equation in slope intercept form</li> <li>• Identify the <math>y</math>-intercept of the table, graph or equation in slope intercept form given a real-world context</li> <li>• Identify the slope from a table</li> <li>• Identify the slope of the table or graph given a real-world context</li> <li>• Identify whether the linear relationship will increase or decrease from a table</li> <li>• Identify whether the linear relationship will increase or decrease from the equation <ul style="list-style-type: none"> <li>• Identify where the linear relationship increases or decreases on a graph</li> <li>• Understand the following concepts and vocabulary: <math>x</math>-axis, <math>y</math>-axis, <math>x</math>-intercept, <math>y</math>-intercept, line, rise, run, fall, slope, rate of</li> </ul> </li> </ul>

	<p>change, table, graph, equation</p> <ul style="list-style-type: none"> <li>Identify what the variables mean in context</li> </ul>
<p><b>MA.8.AR.4 Develop an understanding of two-variable systems of equations.</b></p>	
MA.8.AR.4.1	<p>Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.</p>
	<p><b>Access Point</b></p> <p>MA.8.AR.4.AP.1a Given a system of two linear equations displayed on a graph, identify the solution of a system as the point where the two lines intersect.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Identify the lines on the graph</li> <li>Identify whether the lines intersect</li> <li>Identify the coordinate of the intersection</li> <li>Understand that the solution of a linear system is the point where the lines intersect</li> <li>Understand that some linear systems do not have a solution</li> <li>Understand the following concepts, vocabulary, and symbols: +, -, ×, ÷, =, variable, equation, linear system, intersection, coordinates, coordinate plane, solution, slope, y-intercept, slope-intercept form</li> </ul>
	<p>MA.8.AR.4.AP.1b Identify the coordinates of the point of intersection for two linear equations plotted on a coordinate plane.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Identify the solution to a system (i.e., find when the two lines on the same graph cross)</li> <li>Use manipulatives or tools to identify the solution to the system</li> <li>Understand the following concepts, vocabulary, and symbols: +, -, ×, ÷, =, variable, equation, linear system, intersection, coordinates, coordinate plane, solution, slope, y-intercept, slope-intercept form</li> <li>Identify a coordinate that represents the solution</li> <li>Identify the x-coordinate and y-coordinate of a</li> </ul>

	point on the graph
MA.8.AR.4.2	Given a system of two linear equations represented graphically on the same coordinate plane, determine whether there is one solution, no solution or infinitely many solutions.
	<p><b>Access Point</b></p> <p>MA.8.AR.4.AP.2 Given a system of two linear equations represented graphically on the same coordinate plane, identify whether there is one solution or no solution.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Include defining one solution and no solution</li> <li>• Understand that the solution of the system is the coordinate where the lines intersect</li> <li>• Understand that some systems do not have a solution For example, parallel lines will never intersect; hence, their system would have no solution</li> <li>• Identify whether the lines intersect</li> <li>• Identify the coordinate of the intersection</li> <li>• Manipulate lines on a graph to show no solution (parallel)</li> <li>• Manipulate lines on a graph to show one solution (point of intersection)</li> </ul>
MA.8.AR.4.3	Given a mathematical or real-world context, solve systems of two linear equations by graphing.
	<p><b>Access Point</b></p> <p>MA.8.AR.4.AP.3 given two sets of coordinates for two lines, plot the lines on a coordinate plane and describe or select the solution to a system of linear equations.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Determine whether the lines will intersect</li> <li>• Describe the solution to the system of linear equations <ul style="list-style-type: none"> <li>• Identify the solution to a system (i.e., find when the two lines on the same graph cross) displayed graphically</li> <li>• Select the solution of the system from a list</li> <li>• Graph a line on a coordinate plane when given coordinates</li> <li>• Use manipulatives or tools to graph a line</li> <li>• Use manipulatives or tools to identify a</li> </ul> </li> </ul>



solution to a system of linear equations

- Use the slope and the y-intercept of the line to determine if the lines will intersect
- Understand the following concepts, vocabulary, and symbols: +, -, ×, ÷, =, variable, equation, slope, y-intercept, intersection, coordinates, and coordinate plane

**MA.8.F.1 Define, evaluate and compare functions.**

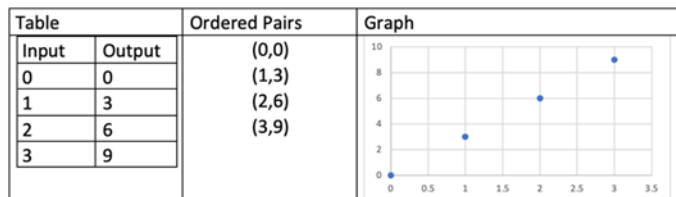
MA.8.F.1.1

Given a set of ordered pairs, a table, a graph or mapping diagram, determine whether the relationship is a function. Identify the domain and range of the relation.

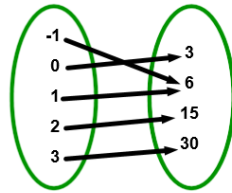
**Access Point**

MA.8.F.1.AP.1a Given a set of ordered pairs, a table or mapping diagram identify whether the relationship is a function.

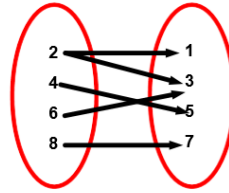
- Identify the input and output values on a T-chart or function table
- Understand that, in a function, an input will only have one output
- Understand that the input values represent the domain, and the output values represent the range
- Using the values of the T-chart, graph the points



- Use the vertical line test to determine whether a relation is a function or non-function
- Identify if the relation is a function given a table
- Identify whether a domain value repeats in the table
- Identify whether a domain value is connected to more than one range value in a mapping



Function



Not a Function

- Understand the following concepts, vocabulary, and symbols: table, mapping diagram, order pairs, function, input, output, vertical line test, domain, range

MA.8.F.1.AP.1b Given a set of ordered pairs, a table or mapping diagram identify the domain and range of the relation.

**Essential Understandings:**

- Understand that the input values represent the domain, and the output values represent the range
- Identify the x coordinate in an ordered pair as an element in the domain
- Identify the y-coordinate in an ordered pair as an element in the range
- Identify the input values, in a table, as the domain and the output values as the range
- Using a mapping, identify the domain as the input values (arrow starts) and the range as the output values (arrow pointing at)
- Use a T-chart and manipulatives to pair the values in the domain to the values in the range
- Use a T-chart to predict the output (y-value)
- Understand the following related vocabulary: domain, range, table, mapping diagram, order pairs, function, input, output, domain, range

MA.8.F.1.2

Given a function defined by a graph or an equation, determine whether the function is a linear function. Given an input-output table, determine whether it could represent a linear function.

**Access Point**

MA.8.F.1.AP.2 Given a function displayed as a graph or an

equation, identify whether the function is a linear function.

**Essential Understandings:**

- Identify a linear function on a graph as one that forms a straight line
- Identify a nonlinear function on a graph as one that does not make a straight line
- Understand the following concepts, vocabulary, and symbols: linear, nonlinear, function, exponent, variable, quadratic
- Label a function on a graph as being either linear or nonlinear
- Identify functions as linear or nonlinear given an equation or graph
- Use tools to identify whether the function displayed on a graph is linear (I.e., Ruler, pipe cleaner, Wikki Stix)
- Identify the exponent on each variable (for example,  $x$  has an exponent of 1,  $x^2$  has an exponent of 2)
- $Y = 2x$ ,  $y = x+5$  (linear equations)
- $Y=x^2$  (quadratic, nonlinear equation)
- Understand that linear functions have the highest exponent of 1

MA.8.F.1.3

Analyze a real-world written description or graphical representation of a functional relationship between two quantities and identify where the function is increasing, decreasing or constant.

**Access Point**

MA.8.F.1.AP.3 Given a functional relationship displayed as a graph, identify where the function is increasing, decreasing or constant.

**Essential Understandings:**

- Identify a graph that is increasing, decreasing and/or constant
- Understand that graphs are read from left to right
- Identify where the graph is increasing or decreasing or constant (Graphingstories.com)
- Define vocabulary in isolation with visual images

- Identify where the function increases or decreases on a graph
- Given a verbal description, determine whether the slope is increasing or decreasing
- Identify characteristics of a graph
- Match a description to a graph
- Use manipulatives to identify where a graph changes direction
- Use manipulatives to represent the relationship between two graphs (E.g., use uncooked spaghetti to describe the slope between an escalator (incline) and an elevator shaft (vertical))
- Identify coordinates (points) on a graph
- Describe the situation that may account for the characteristics in the graph

### **Geometric Reasoning**

***MA.8.GR.1 Develop an understanding of the Pythagorean Theorem and angle relationships involving triangles.***

MA.8.GR.1.  
1

Apply the Pythagorean Theorem to solve mathematical and real-world problems involving unknown side lengths in right triangles.

**Access Point**

MA.8.GR.1.AP.1 Find the hypotenuse of a two-dimensional right triangle using the Pythagorean Theorem.

**Essential Understandings:**

- Identify the formula for the Pythagorean Theorem
- Understand that every right triangle has exactly one right angle (90 degrees)
- Identify what each variable in the Pythagorean Theorem represents
- Label the legs and the hypotenuse of the given figure
- Measure the lengths of the legs and the hypotenuse of the given figure
- Use a graphic organizer to organize the measurements of the legs and hypotenuse, using appropriate tools as needed

	<ul style="list-style-type: none"> <li>• Use tools to find the square and square root of a number</li> <li>• Use substitution or a graphic organizer to find the value of the missing side length</li> <li>• Understand the following concepts and vocabulary: Pythagorean Theorem, length, right triangle, hypotenuse, leg, and angle</li> </ul>
<p>MA.8.GR.1.2</p>	<p>Apply the Pythagorean Theorem to solve mathematical and real-world problems involving the distance between two points in a coordinate plane.</p> <p><b>Access Point</b></p> <p>MA.8.GR.1.AP.2 Given the Pythagorean Theorem, determine lengths/distances between two points in a coordinate system by forming right triangles, with natural number side lengths.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Use the given two points to form a right triangle</li> <li>• Understand that the length of a diagonal line may be different than the number of blocks it intersects</li> <li>• Use the coordinate grid to count the measure of each side length of the drawn triangle</li> <li>• Identify the formula for the Pythagorean Theorem</li> <li>• Identify what each variable in the Pythagorean Theorem represents</li> <li>• Label the legs and the hypotenuse of the given figure</li> <li>• Measure the lengths of the legs and the hypotenuse of the given figure</li> <li>• Use substitution or a graphic organizer to calculate a missing side using the Pythagorean Theorem, using appropriate tools as needed</li> <li>• Enter information into the formula for the Pythagorean Theorem to solve problems</li> <li>• Understand the following concepts and vocabulary: Pythagorean Theorem, length, right triangle, legs, hypotenuse, and angle</li> </ul>
<p>MA.8.GR.1.3</p>	<p>Use the Triangle Inequality Theorem to determine if a triangle can be formed from a given set of sides. Use the</p>

converse of the Pythagorean Theorem to determine if a right triangle can be formed from a given set of sides.

**Access Point**

MA.8.GR.1.AP.3a Measure the sides of triangles to establish facts about the Triangle Inequality Theorem (i.e., the sum of two side lengths is greater than the third side).

**Essential Understandings:**

- Identify the formula for the Triangle Inequality Theorem
- Identify what each variable in the Triangle Inequality Theorem represents
- Label the legs and the hypotenuse of the given figure
- Measure the lengths of the legs and the hypotenuse of the given figure
- Use substitution or a graphic organizer to organize the measurements of the legs and hypotenuse, using appropriate tools as needed
- Use tools (calculator, number line, graphic organizer, etc.) to determine whether the sum of the legs is greater than the hypotenuse
- Understand the following concepts and vocabulary: Pythagorean Theorem, Triangle Inequality Theorem, length, right triangle, hypotenuse, leg, and angle

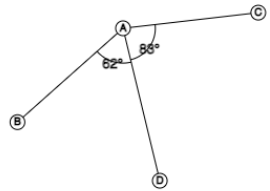
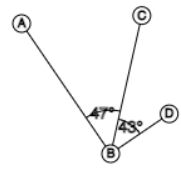
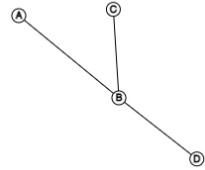
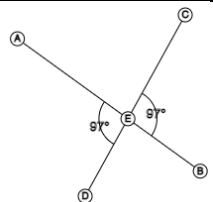
MA.8.GR.1.AP.3b Substitute the side lengths of a given figure into the Pythagorean Theorem to determine if a right triangle can be formed.

**Essential Understandings:**

- Identify the formula for the Pythagorean Theorem
- Identify what each variable in the Pythagorean Theorem represents
- Label the legs and the hypotenuse of the given figure
- Measure the lengths of the legs and the hypotenuse of the given figure
- Use tools to find the square and square root of a number.
- Use substitution or a graphic organizer to organize the measurements of the legs and

	<p>hypotenuse, using appropriate tools as needed</p> <ul style="list-style-type: none"> <li>• Use tools (calculator, number line, graphic organizer, etc.) to determine whether the sum of the legs is greater than the hypotenuse</li> <li>• Identify whether the triangle is a right triangle based on its measurements</li> <li>• Understand that every right triangle has exactly one right angle (90 degrees)</li> <li>• Understand the following concepts and vocabulary: equality, square, square root, Pythagorean Theorem, length, right triangle, hypotenuse, leg, and angle</li> </ul>
<p>MA.8.GR.1.4</p>	<p>Solve mathematical problems involving the relationships between supplementary, complementary, vertical or adjacent angles.</p> <p><b>Access Point</b></p> <p>MA.8.GR.1.AP.4 Identify supplementary, complementary, vertical or adjacent angle relationships.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given an angle measure, draw an angle</li> <li>• Recognize that the angle measure of a straight line is 180 degrees</li> <li>• Use a protractor to measure the missing angle</li> <li>• Understand the following concepts and vocabulary: acute, obtuse, right, straight-line, transversal, vertical angles, corresponding angles, alternate interior angles, supplementary angles</li> <li>• Match or identify angle measurements</li> <li>• Describe angles and parallel lines using their characteristics, i.e., size, sides, lines, and angle measures</li> <li>• Use appropriate tools as needed</li> <li>• Use addition or subtraction to determine the missing angle measurement. (E.g., Angle ABC and Angle EDG are complementary. Angle ABC measures 35 degrees. Angle EDG has to measure 55 degrees because <math>90-35=55</math>)</li> <li>• Identify vertical angles, corresponding angles, and alternate interior angles. Understand that</li> </ul>

the angles are congruent

Relationship	Characteristics	Image
Adjacent	have a common vertex and a common side but do not overlap	
Complementary	The sum of the two angle measurements equals 90 degrees	
Supplementary	The sum of two angles whose sum equals 180 degrees *forms a straight line	
Vertical	each of the pairs of opposite angles made by two intersecting lines	

MA.8.GR.1.5

Solve problems involving the relationships of interior and exterior angles of a triangle

**Access Point**

MA.8.GR.1.AP.5 Given an image, solve simple problems involving the relationships of interior and exterior angles of a triangle.

**Essential Understandings:**

- Identify the interior angles of a triangle
- Identify the exterior angles of a triangle
- Understand that an interior angle and its exterior angle create a linear pair with a supplementary angle relationship
- Given a triangle, measure each angle
- Given a triangle, measure the angle with a missing measure using a tool, i.e., protractor



- Given a triangle, tear the angles off and put them together to make a straight line
- Given a triangle, subtract numbers from 180 using a table.

Angle A	Angle B	Angle A + Angle B	180 - (Angle A + B) = Angle C	Angle A + Angle B + Angle C = 180

- Use addition or subtraction to determine the missing angle measurement in triangles. (E.g., Angle A = 60 degrees, Angle B = 40 degrees, Angle A + Angle B = 100 degrees, therefore Angle C = 180 - 100 = 80 degrees)
- Given a linear pair, measure the angle with the missing measurement, using a tool, i.e., protractor, virtual manipulative, etc.
- Given a triangle, use a ruler to construct an exterior angle.
- Recognize that a triangle consists of three angles that total 180 degrees
- Recognize that the angle measure of a straight line is 180 degrees
- Understand the following concepts and vocabulary: acute, obtuse, right, straight line, supplementary angles, exterior angle, interior angle, supplementary, protractor,
- Match or identify angle measurements
- Describe triangles by telling about their shape, sides, lines, and angles

MA.8.GR.1.6

Develop and use formulas for the sums of the interior angles of regular polygons by decomposing them into triangles.

**Access Point**

MA.8.GR.1.AP.6 Use tools to calculate the sum of the interior angles of regular polygons when given the formula.

**Essential Understandings:**

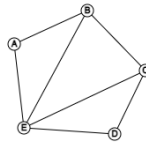
- Identify the number of sides in a polygon
- Identify the number of sides in common

polygons by name (square, rectangle, quadrilateral, pentagon, octagon, etc.)

- Understand that a regular polygon has sides and angles of equal measure
- Understand how to use the formula for the sum of interior angles:

Sum of interior angles =  $180(n - 2)$ , where  $n$  is the number of sides

- Given a polygon, use a single vertex to construct triangles within the figure



- Understand that the sum of the interior angles in a triangle is 180 degrees
- Identify parts of a polygon (interior angles, sides, vertices)

**MA.8.GR.2 Understand similarity and congruence using models and transformations.**

MA.8.GR.2.  
1

Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.

**Access Point**

MA.8.GR.2.AP.1 Given two figures on a coordinate plane, identify if the image is translated, rotated or reflected.

**Essential Understandings:**

- Understand the following concepts and vocabulary: transformation, image, pre-image, 90-degree rotation, 180-degree rotation, 270-degree rotation, 360-degree rotation, clockwise, counter-clockwise, vertical, horizontal, x-axis, y-axis, origin, congruent, rotation, reflection, and translation
- Model a rotation, reflection, and translation on the coordinate plane using manipulatives
- Identify a rotation, reflection, and translation when it occurs on the coordinate plane

	<ul style="list-style-type: none"> <li>• Use manipulatives to demonstrate translations (sliding object)</li> <li>• Use manipulatives to demonstrate rotation (rotating figure)</li> <li>• Use manipulatives to demonstrate reflections (flipping object)</li> <li>• Trace a figure and slide it over to translate the figure</li> <li>• Trace a figure and rotate it around the origin to create a rotation</li> <li>• Trace a figure and reflect it across the x-axis or y-axis to create a reflection</li> <li>• Understand that a translation, rotation, or reflection does not change the size of the shape, only its positioning</li> <li>• Understand that an image and its pre-image should be congruent for translations, reflections, and rotations</li> </ul>
<p>MA.8.GR.2.2</p>	<p>Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.</p> <p><b>Access Point</b></p> <p>MA.8.GR.2.AP.2 Given a preimage and image describe the effect the dilation has on the two figures.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following concepts and vocabulary: transformation, dilation, image, pre-image, scale drawing, scale factor, dimension, enlarge, reduce clockwise, counterclockwise, vertical, horizontal, x-axis, y-axis, origin, rotation, reflection, and translation</li> <li>• Identify when shapes are similar, but different sizes (e.g., show the same shape at various sizes – 25%, 50%, 150% – when presented with two-dimensional or three-dimensional shapes)</li> <li>• Use different size manipulatives of the same figure to demonstrate dilations</li> </ul>



- Use the zoom in/zoom out feature on a copier or computer screen to demonstrate the increasing and/or decreasing size of an image
- Understand that a dilation does not change the shape itself, only its size
- Understand that the pre-image and its image will be similar, but may not be congruent
- Understand that multiplying makes an object bigger, and dividing (or multiplying by a fraction) makes an object smaller
- Use appropriate tools to solve a one-step equation
- Understand that a scale factor of 1 will create a congruent figure
- Understand scale factors greater than 1 will produce a larger image
- Understand that scale factors smaller than 1 will produce a smaller image

MA.8.GR.2.  
3

Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.

**Access Point**

MA.8.GR.2.AP.3 Identify the coordinates of the vertices of a common polygon after a single translation, rotation or dilation on the coordinate plane.

**Essential Understandings:**

- Understand the following vocabulary: vertex, side, dilation, reflection, rotations, translation, common polygon, coordinate, coordinate plane, scale factor, increasing and decreasing of size and scale
- Use manipulatives to demonstrate rotations, reflections, or translations
- Match or identify when a two-dimensional drawing has been dilated, rotated, reflected, or translated

- Trace a figure (pre-image) on a coordinate plane and slide it over to translate the figure. Identify the coordinates of the vertices in its new position (image)
- Trace a figure (pre-image) on a coordinate plane and rotate it to create a rotation

Identify the coordinate of the vertices (image) in its new position

- Trace a figure (pre-image) on a coordinate plane and reflect it across either the x-axis or y-axis to create a reflection

Identify the coordinates of the vertices in its new position (image)

- Given a figure on a coordinate plane, identify the coordinates of its vertices
- Using manipulatives identify two figures that are the same shape and size
- Using manipulatives identify two figures that are different sizes but the same shape. Use the two figures to find the coordinates of the vertices
- Draw two figures that are different sizes but the same shape on a coordinate plane. Identify the coordinates of each figure

MA.8.GR.2.  
4

Solve mathematical and real-world problems involving proportional relationships between similar triangles.

**Access Point**

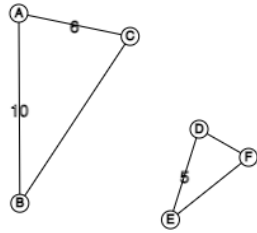
MA.8.GR.2.AP.4 Use tools to solve mathematical problems using proportions between similar triangles.

**Essential Understandings:**

- Understand that similar triangles will have congruent angle measures, but may not have congruent side lengths
- Select two triangles that are the same shape
- Select two objects that have different shapes
- Use appropriate tools as needed to duplicate a shape (e.g., wiki sticks, computers, interactive white boards, markers, transparency film, patty paper, sheet protectors)
- Given two shapes, label (identify, point to,

mark,) the corresponding parts of congruent figures

- Given two shapes, label (identify, point to, mark,) the corresponding parts of similar figures
- Describe the characteristics of two figures that are the same
- Describe the characteristics of two figures that are different (For example, in the figure below side A is twice the length of side a)



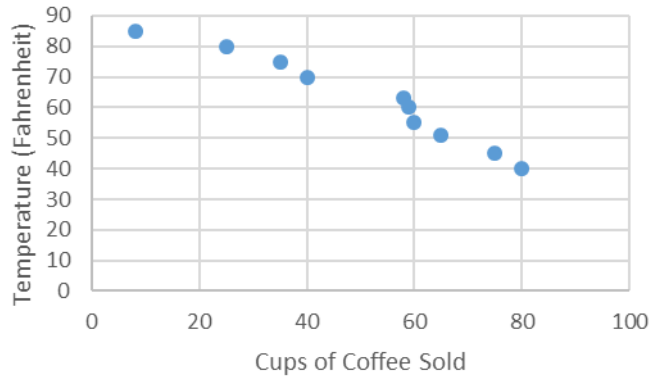
- Understand the following concepts and vocabulary: similar, congruent, angles, corresponding, proportional, and transformation
- Identify vertices on a graph for each figure
- Identify vertices on a graph to determine the length of a leg of the triangle
- Identify the measures of the side lengths of a triangle
- Match a triangle with its proportional relationship
- Use an applet to manipulate similar and congruent figures
- Use a calculator to determine whether the sides of two figures are proportional
- Use a graphic organizer to determine whether the side lengths of two figures are proportional

Figure	
	$\frac{\text{big}}{\text{small}} :$ $\frac{A}{a} = \frac{B}{b}$

		$\frac{AC}{DF} = \frac{AB}{DE}$ $\frac{6}{3} = \frac{10}{5}$ $10 (DF) = 6 * 5$ $10 (DF) = 30$ $DF = 3$
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### Data Analysis and Probability

<b>MA.8.DP.1 Represent and investigate numerical bivariate data</b>	
MA.8.DP.1.1	<p>Given a set of real-world bivariate numerical data, construct a scatter plot or a line graph as appropriate for the context.</p> <p><b>Access Point</b></p> <p>MA.8.DP.1.AP.1 Graph bivariate data using a scatter plot.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Locate points on the x-axis and y-axis on an adapted grid (not necessarily numeric)</li> <li>• Understand the following concepts and vocabulary: best fit line, variable, outliers, positive association, negative association, no association, bivariate, scatter plot</li> <li>• Graph a series of data points on a coordinate grid</li> <li>• Identify what the x-axis and y-axis represent</li> <li>• Enter data into a graph using manipulatives, as needed</li> <li>• Understand basic information from simple graphs (e.g., interpret a scatter plot using the understanding that each point on the graph represents a relationship between two different variables). For example, the company sold \$520 worth of ice cream when it was 22 degrees Celsius outside</li> </ul>



MA.8.DP.1.2

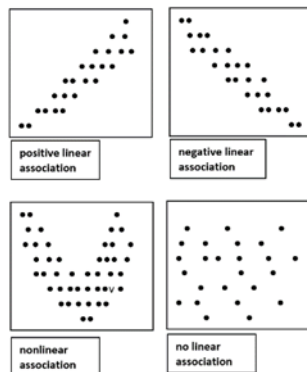
Given a scatter plot within a real-world context, describe patterns of association.

**Access Point**

MA.8.DP.1.AP.2 Given a scatter plot, identify whether the patterns of association are no association, positive association, negative association, linear or nonlinear.

**Essential Understandings:**

- Identify a similar distribution when given a choice of three (e.g., when shown a positive association, can select a second example of a positive association from three choices).
- Identify the associations between the variables using supports (E.g., use a template to determine the association, use a pre-made scatter plot transparency and place on top of a given scatter



- Locate points on the x-axis and y-axis on an adapted grid (not necessarily numeric).
- Describe, in general, the direction of the points on the grid (for example, the points are increasing from left to right)



- Understand the following concepts and vocabulary

MA.8.DP.1.3

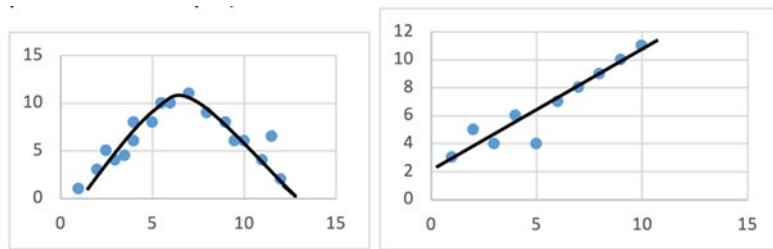
Given a scatter plot with a linear association, informally fit a straight line.

**Access Point**


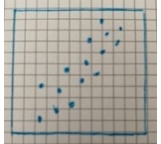
MA.8.DP.1.AP.3 Given a scatter plot with a linear association, use tools to draw or place a line of best fit.

**Essential Understandings:**

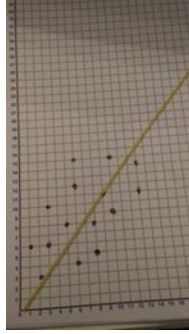
- Draw a line between two points on a graph.
- Draw a line between at least two points on the graph that approximates the general direction of the data
- Identify linear and non-linear associations in various scatter plots. For example:



- Identify when data points are close together or spread out (strong or weak association). For example:

Strong Positive Linear Association	Weak Positive Linear Association
	

- Use appropriate tools (uncooked spaghetti noodle, clear ruler, Popsicle stick, etc.) to visually approximate the line of best fit.



- Understand the following concepts and vocabulary: best fit line, variable, outliers, scatter plots, data points, linear associations, nonlinear associations.
- Given three choices, select the line of best fit.



**MA.8.DP.2 Represent and find probabilities of repeated experiments.**

MA.8.DP.2.1

Determine the sample space for a repeated experiment.

**Access Point**

MA.8.DP.2.AP.1 Use a tool (table, list or tree diagram) to record results of a repeated experiment.

**Essential Understandings:**

- Record the result of an experiment
- Use a tree diagram to record the result (outcome) of a repeated experiment
- Use a table to record the result (outcome) of a repeated experiment
- Use a chart to capture the outcomes of coin flips or dice rolls
- Use items like coins to generate outcomes for a repeated experiment
- Use coins to represent the theoretical probability for a sample space – show rows of two coins each, one with heads face up and

	<p>one with tails up; each row represents another stage of the experiment</p> <ul style="list-style-type: none"> <li>• Using manipulatives and a chart to capture the outcomes of coin flips or dice rolls</li> <li>• Given a set of items, identify items that are in the set and items that are not in the set</li> <li>• Understand the following concepts, symbols, and vocabulary: probability, likelihood, experiment, outcome, event, chance event, compound event, simple event</li> </ul>
MA.8.DP.2.2	<p>Find the theoretical probability of an event related to a repeated experiment.</p> <p><b>Access Point</b></p> <p>MA.8.DP.2.AP.2 Select the theoretical probability of an event from a list.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Given a set of items, identify items that are in the set and items that are not in the set</li> <li>• Identify the difference between theoretical and experimental probability</li> <li>• Match a scenario with its theoretical probability</li> <li>• Identify the possible outcomes of an experiment</li> <li>• Given a set of items, identify the probability of selecting a specific item from the set</li> <li>• Use a model to identify the probability of a chance event.</li> <li>• Understand the value for probability of a chance event ranges between 0 and 1</li> <li>• Identify the formula for finding theoretical probability of an event (theoretical probability = number of ways it can happen/total number of outcomes)</li> <li>• Use items like coins to determine the probability of an outcome (1/2 heads)</li> <li>• Understand the following concepts, symbols, and vocabulary: probability, likelihood, experiment, outcome, event, chance event,</li> </ul>

	compound event, simple event
MA.8.DP.2.3	Solve real-world problems involving probabilities related to single or repeated experiments, including making predictions based on theoretical probability.
	<p><b>Access Point</b></p> <p>MA.8.DP.2.AP.3 Compare actual results of an experiment with its theoretical probability (e.g., make a statement that describes the relationship between the actual results of an experiment with its theoretical probability [e.g., more, less, same, different, equal]).</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Identify the formula for finding theoretical probability of an event (theoretical probability = number of ways it can happen/total number of outcomes)</li> <li>• Identify the formula for finding the experimental probability of an event (Experimental Probability = number of ways it actually happened/total number of outcomes)</li> <li>• Identify the theoretical probability of an experiment (theoretical probability = number of ways it can happen/total number of outcomes)</li> <li>• Identify the experimental probability of an experiment (Experimental Probability = number of ways it actually happened/total number of outcomes)</li> <li>• Identify the characteristics of the theoretical and experimental probability</li> <li>• Compare and describe the theoretical and experimental probability of an experiment</li> <li>• Use the recorded results from an experiment to describe the experimental probability of a chance event</li> <li>• Use the results from an experiment to describe the theoretical probability of a chance event</li> <li>• Identify the similarities and differences between theoretical and experimental probability</li> <li>• Identify the correct match for probabilities and</li> </ul>

results. e.g., Select a picture card that describes the relationship between the actual results of an experiment with its theoretical probabilities (e.g., more, less, same, different, equal)

- Understand the following concepts, symbols, and vocabulary: probability, likelihood, experiment, outcome, event, chance event, compound event, simple event

### 9-12 Number Sense and Operations Strand

***MA.912.NSO.1 Generate equivalent expressions and perform operations with expressions involving exponents, radicals or logarithms.***

MA.912.NSO.1.1

Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.

**Access Point**

MA.912.NSO.1.AP.1 Evaluate numerical expressions involving rational exponents.

**Essential Understandings:**

- Understand the following concepts, symbols, and/or vocabulary for: numerator, denominator, expression, exponent, negative exponent, radical expression, raising to a power
- Understand the parts of a fractions.
- Understand that a rational number can be represented by a fraction
- Identify expressions with exponents
- Create a model with objects to show that the exponent of a number says how many times to multiply the number by itself  
e.g., substitute two chips for each "2"  
Exp.  $2^3 = 2 \times 2 \times 2 = 8$

	<ul style="list-style-type: none"> <li>• Understand that a negative exponent will result in a fraction with a numerator of 1 (e.g., <math>5^{-2} = \frac{1}{5^2} = \frac{1}{25}</math>)</li> <li>• Understand that a fractional exponent is another way to write a radical expression (e.g., <math>16^{\frac{1}{2}} = \sqrt{16} = 4</math>; <math>27^{\frac{1}{3}} = \sqrt[3]{27} = 3</math>)</li> </ul>
MA.912.NSO.1.2	<p>Generate equivalent algebraic expressions using the properties of exponents.</p> <p><b>Access Point</b> MA.912.NSO.1.AP.2 Identify equivalent algebraic expressions using properties of exponents.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following vocabulary: algebraic expression, exponents, simplest form, variable, base number, integers.</li> <li>• Understand addition, subtraction, multiplication, and division of integers.</li> <li>• Identify the parts of an algebraic expression. e.g., <math>x^7</math> where <math>x</math> is the base number and 7 is the exponent.</li> <li>• Create a model with objects to represent an algebraic expression. e.g., substitute manipulatives or algebra tiles for each "a" e.g., <math>a^7 = a \times a \times a \times a \times a \times a \times a</math> <math>a = aaaaaaa</math>.</li> <li>• Identify expressions with exponents e.g., <math>(x^4)(x^3)</math></li> <li>• Understand the properties of exponents.</li> <li>• Use the properties of exponents to simplify algebraic expressions.</li> </ul>
MA.912.NSO.1.3	<p>Generate equivalent algebraic expressions involving radicals or rational exponents using the properties of exponents.</p>
	<p><b>Access Point</b> MA.912.NSO.1.AP.3 Using properties of exponents, identify equivalent algebraic expressions involving radicals and rational exponents. Radicands are limited to monomial algebraic expression.</p>

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following concepts, symbols, and vocabulary: base number, exponent, integer, variable, monomial algebraic expression, radical exponents, rational exponents, equivalent, radicands.</li> <li>• Add, subtract, and multiply integers (e.g., use manipulatives, a number line or calculator to add <math>2 + -5</math>).</li> <li>• Add, subtract, and multiply fractions (e.g., use manipulatives, online tools).</li> <li>• Understand what the exponent represents in expanded form. (e.g., <math>8^3 = 8 \times 8 \times 8</math>).</li> <li>• Understand the following properties of exponents: <ul style="list-style-type: none"> <li>Quotient Rule</li> <li>Product Rule</li> <li>Power of a Power Rule</li> <li>Power of a Product Rule</li> <li>Power of a Rule</li> <li>Zero Exponent Rule</li> <li>Negative Exponent Rule</li> </ul> </li> </ul>
MA.912.NSO.1.4	<p>Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.</p> <p><b>Access Point</b></p> <p>MA.912.NSO.1.AP.4 Apply previous understanding of operations with rational numbers to add and subtract numerical radicals that are in radical form.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand vocabulary: addition, subtraction, expression, rational numbers, irrational numbers, radical numbers.</li> <li>• Add and subtract integers (e.g., use manipulatives, a number line or calculator to add <math>2 + -5</math>)</li> <li>• Add and Subtract fractions.</li> <li>• Recognize the difference between rational numbers and irrational numbers.</li> <li>• Recognize the difference between a rational number in radical form and a rational number not in radical form.</li> <li>• Understand adding and subtracting rational numbers in radical form follows the same rules</li> </ul>

	as adding and subtracting variables.
MA.912.NSO.1.5	Add, subtract, multiply and divide algebraic expressions involving radicals.
	<b>Access Point</b> MA.912.NSO.1.AP.5 Add and subtract algebraic expressions involving radicals. Radicands are limited to monomial algebraic expressions.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand vocabulary: addition, subtraction, expression, radical expressions, radicands, monomial algebraic expressions.</li> <li>• Add and subtract integers (e.g., use manipulatives, a number line or calculator to add <math>2 + -5</math>).</li> <li>• Add and subtract algebraic expressions.</li> <li>• Recognize the difference between algebraic expressions in radical form and not in radical form.</li> <li>• Adding and subtracting radical expressions follows the same rules as adding and subtracting variables</li> </ul>
MA.912.NSO.1.6	Given a numerical logarithmic expression, evaluate and generate equivalent numerical expressions using the properties of logarithms or exponents.
	<b>Access Point</b> MA.912.NSO.1.AP.6 Given a numerical logarithmic expression, identify an equivalent numerical expression using the properties of logarithms or exponents.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand the concepts and vocabulary: numerical logarithmic expression, exponent, equivalent, logarithm, base, properties.</li> <li>• Identify expressions with exponents. E.g., <math>(2^4)</math>.</li> <li>• Understand what the exponent represents in expanded form. (e.g., <math>24 = 2 \times 2 \times 2 \times 2</math>)</li> <li>• Understand what question a logarithm asks.</li> <li>• Use the properties of logarithms to rewrite the expression.</li> <li>• Product rule</li> </ul>



	<ul style="list-style-type: none"> <li>• Quotient rule</li> <li>• Power rule</li> <li>• Change of base rule</li> <li>• Equality rule</li> </ul>
MA.912.NSO.1.7	<p>Given an algebraic logarithmic expression, generate an equivalent algebraic expression using the properties of logarithms or exponents.</p> <p><b>Access Point</b>  MA.912.NSO.1.AP.7 Given an algebraic logarithmic expression, identify an equivalent algebraic expression using the properties of logarithms or exponents.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the concepts and vocabulary: numerical logarithmic expression, exponent, equivalent, logarithm, base, properties.</li> <li>• Identify expressions with exponents. E.g., <math>(2^4)</math>.</li> <li>• Understand what the exponent represents in expanded form. (e.g., <math>24 = 2 \times 2 \times 2 \times 2</math>)</li> <li>• Understand what question a logarithm asks.</li> <li>• Use the properties of logarithms to rewrite the expression.</li> <li>• Product rule</li> <li>• Quotient rule</li> <li>• Power rule</li> <li>• Change of base rule</li> <li>• Equality rule</li> </ul>
MA.912.NSO.2.1	<p>Extend previous understanding of the real number system to include the complex number system. Add, subtract, multiply and divide complex numbers.</p> <p><b>Access Point</b>  MA.912.NSO.2.AP.1 Extend previous understanding of the real number system to include the complex number system. Add and subtract complex numbers.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Extend previous understanding of the real number system to include the complex number system. Add and subtract complex numbers.</li> <li>• Essential Understandings</li> </ul>

- Understand the following concepts and vocabulary: real number system, complex number system, complex numbers, real numbers, coefficient.
- Understand how to add and subtract real numbers.
- Understand the parts of a complex number, real and imaginary.
- Understand when adding and subtracting complex numbers, the real part of the complex number can be added and subtracted together, and the imaginary part or the complex number can be added and subtracted together Ex.  $5+2i-3+4i=2+6i$

### 9-12 Algebraic Reasoning Strand

#### ***MA.912.AR.1 Interpret and rewrite algebraic expressions and equations in equivalent forms.***

MA.912.AR.1.1

Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.

#### **Access Point**

MA.912.AR.1.AP.1 Identify a part(s) of an equation or expression and explain the meaning within the context of a problem.

#### **Essential Understandings:**

- Understand the following concepts and vocabulary: equation, expression, add (+), subtract (-), multiply (x), divide ( $\div$ ), equal (=), Greater than (>), Less than (<), unknown (x), variables, and real-world context
- Understand in a problem with real world context, the variables have meaning within the context of the problem.  
 Ex. Distance Problem  
 Distance Formula:  $d=rt$  (d = distance, r = rate, t = time)  
  
 Ex. Interest Problem  
 Interest Formula:  $I = Prt$  (I = interest, P = principal, r = rate, t = time in years)

	<p>Ex Match items from a problem with variables (e.g., In the expression <math>6x + 7y</math>, students explain that Bill had 6 times as many apples and 7 times as many oranges as Sam, with <math>x</math> representing the number of apples and <math>y</math> representing the number of oranges)</p>
<p>MA.912.AR.1.2</p>	<p>Rearrange equations or formulas to isolate a quantity of interest.</p> <p><b>Access Point</b>  MA.912.AR.1.AP.2 Rearrange an equation or a formula for a specific variable.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following concepts and vocabulary: variable, symbol, equation, multivariate equation, add (+), subtract (-), multiply (<math>\times</math>), divide (<math>\div</math>), equal (=), unknown, formulas,</li> <li>• Understand when rearranging an equation, isolate for variable of interest.  Ex. <math>d=rt</math> (<math>d</math> = distance, <math>r</math> = rate, <math>t</math> = time)  Solve for <math>t</math></li> <li>• Understand algebraic rules (e.g., what you do to one side of the equation you must do to the other).  Ex. Distance Formula: <math>d=rt</math> (<math>d</math> = distance, <math>r</math> = rate, <math>t</math> = time)  Solve for <math>t</math>  <math>d = rt</math>  Divide <math>r</math> on both sides  <math>\frac{d}{r} = \frac{rt}{r}</math>  <math>\frac{d}{r} = t</math></li> </ul> <p>Ex. Interest Formula: <math>I = Prt</math> (<math>I</math> = interest, <math>P</math> = principal, <math>r</math> = rate, <math>t</math> = time in years)  Solve for <math>P</math>  <math>I = Prt</math>  Divide <math>rt</math> on both sides  <math>\frac{I}{rt} = \frac{Prt}{rt}</math>  <math>\frac{I}{rt} = P</math></p>

MA.912.AR.1.3	<p>Add, subtract and multiply polynomial expressions with rational number coefficients.</p> <p><b>Access Point</b> MA.912.AR.1.AP.3 Add, subtract and multiply polynomial expressions with integer coefficients.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following vocabulary and symbols: polynomial, variable, exponent, constant, coefficient, and like terms.</li> <li>• Identify examples of polynomials (an expression consisting of variables and coefficients with non-negative exponents).</li> <li>• Identify non-examples of polynomials.</li> <li>• Sort variables into like terms when adding and subtracting polynomials (e.g., sort all the x's and y's).</li> </ul> <p>Ex.</p> $  \begin{array}{r}  x^2 + 3y - 2xy + 4x - 5x^2 + 10y - 18xy + 7x \\  x^2 - 5x^2 - 2xy - 18xy + 7x + 4x + 3y + 10y \\  - 4x^2 - 20xy + 11x + 13y  \end{array}  $ <ul style="list-style-type: none"> <li>• Understand that polynomials can be added, subtracted, and multiplied (multiplication should be limited to no more than two polynomials).</li> </ul>
MA.912.AR.1.4	<p>Divide a polynomial expression by a monomial expression with rational number coefficients.</p> <p><b>Access Point</b> MA.912.AR.1.AP.4 Divide a polynomial expression by a monomial expression with integer coefficients.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following vocabulary and symbols: polynomial expression, monomial expression, variable, exponent, constant, coefficient, numerator, denominator, simplify, and distributive property.</li> <li>• Understand that the monomial in the numerator is divided by a monomial in the denominator.</li> </ul>

Ex.  $\frac{4b}{2b} = 2$

- Understand that the denominator must be distributed to every term in the numerator.

Ex.

$$\frac{12x^2 - 10x + 1}{2x} = \frac{12x^2}{2x} - \frac{10x}{2x} + \frac{1}{2x}$$

- Understand that the terms need to be simplified.

Ex.

$$\frac{12x^2}{2x} - \frac{10x}{2x} + \frac{1}{2x} = 6x - 5 + \frac{1}{2x}$$

MA.912.AR.1.5

Divide polynomial expressions using long division, synthetic division or algebraic manipulation. Related Access Points

**Access Point**

MA.912.AR.1.AP.5 Divide polynomial expressions using long division, synthetic division, and algebraic manipulation where the denominator is a linear expression.

**Essential Understandings:**

- Understand the following related vocabulary: numerator, denominator, fraction, variable, polynomial, factoring, division, divisor, dividend, quotient, remainder, synthetic division, linear.
- Understand factoring polynomials.
- Understand that manipulatives can be used to factor an equation, Ex, algebra tiles.
- Understand that an equation can be simplified by crossing out similar factors in the numerator and denominator (e.g.,  $ab/ac=b/c$  ).
- Understand that manipulatives can be used to model dividing polynomials.
- Understand that synthetic division can be used to divide polynomials only when the divisor is linear.
- Understand that a rational expression can be rewritten using long division.

MA.912.AR.1.6	Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of polynomials.
	<p><b>Access Point</b></p> <p>MA.912.AR.1.AP.6 Solve mathematical or real-world problems involving addition, subtraction, multiplication or division of polynomials with integer coefficients.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following concepts and vocabulary: add (+), subtract (-), multiply (x), divide (÷), equal (=), unknown, polynomials, integers, distributive property, numerator, denominator, variable, equation, factor.</li> <li>• Understand how to add, subtract, multiply, and divide integers. (limited to 2-digit numbers).</li> <li>• Sort variables into like terms when adding and subtracting polynomials (e.g., sort all the x's and y's).</li> </ul> <p style="padding-left: 40px;">Ex.</p> $x^2+3y-2xy+4x-5x^2+10y-18xy+7x$ $x^2-5x^2-2xy-18xy+7x+4x+3y+10y-4x^2-20xy+11x+13y$ <ul style="list-style-type: none"> <li>• Understand that multiplying polynomials requires distributive property (limited to no more than two polynomials).</li> <li>• Understand that manipulatives can be used to factor an equation, Ex, algebra tiles.</li> <li>• Understand that an equation can be simplified by crossing out similar factors in the numerator and denominator (e.g., <math>ab/ac=b/c</math> ).</li> <li>• Understand that manipulatives can be used to model addition, subtraction, multiplication and division of polynomials.</li> </ul>
MA.912.AR.1.7	<p>Rewrite a polynomial expression as a product of polynomials over the real number system.</p> <p><b>Access Point</b></p> <p>MA.912.AR.1.AP.7 Factor a quadratic expression.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following concepts and vocabulary: factor, coefficient, integer, terms, exponent, base, constant, variable,</li> </ul>

	<p>binomial, monomial, polynomial, multiplication, division, quadratic</p> <ul style="list-style-type: none"> <li>• Understand how to multiply integers (using tools)</li> <li>• Understand how to divide integers (using tools)</li> <li>• List the factors of integers. (using tools)</li> <li>• Ex.</li> <li>• 24</li> <li>• Factors: (2)(12); (3)(8); (4)(6); (1)(24)</li> <li>• Understand that factoring a quadratic expression will result in the product of monomials and/or binomials.</li> <li>• Ex.</li> <li>• Monomial and binomial: <math>4x^2 + 2x = 2x(2x + 1)</math></li> <li>• Two binomials: <math>x^2 + 5x + 6 = (x + 2)(x + 3)</math></li> <li>• Use factoring tools/methods to factor quadratic equations (e.g., Algebra tiles, guess and check, quadratic formula, order the steps, etc.)</li> </ul>
MA.912.AR.1.8	<p>Rewrite a polynomial expression as a product of polynomials over the real or complex number system.</p> <p><b>Access Point</b></p> <p>MA.912.AR.1.AP.8 Select a polynomial expression as a product of polynomials with integer coefficients over the real or complex number system.</p>

**Essential Understandings:**

- Understand the following terms and vocabulary: integers, polynomials, multiplication, term, complex number system, distributive property, monomials, binomials, factors, factoring, quadratic expressions, polynomial expressions.
- Understand how to list the factors of integers (using tools)

Ex. 24

Factors: (2)(12); (3)(8); (4)(6); (1)(24)

- Understand how to multiply integers.
- Understand that a polynomial expression is an expression consisting of more than one term.
- Understand that multiplying polynomials requires distributive property (Limited to no more than two polynomials)
- Understand that a complex number is in the form of  $a + bi$
- Understand the following rules for the complex number system:

$$i^1 = i$$

$$i^2 = -1$$

- Understand that factoring a quadratic expression will result in the product of monomials and/or binomials.

Ex. Monomial and binomial:  $4x^2 + 2x = 2x(2x + 1)$

- Understand how to use factoring tools/methods to factor quadratic equations (E.g., Algebra tiles, guess and check, quadratic formula, order the steps, etc.)

MA.912.AR.1.9

Apply previous understanding of rational number operations to add, subtract, multiply and divide rational algebraic expressions.

**Access Point**

MA.912.AR.1.AP.9 Apply previous understanding of rational number operations with common denominators to add and subtract rational expressions.



**Essential Understandings:**

- Understand the following terms and vocabulary: numerator, denominator, rational expression, fraction, polynomials, common denominator, simplify, addition, subtraction
- Understand how to simplify a fraction with whole numbers
- Understand how to add or subtract polynomial expressions
- Understand that a rational expression is a fraction where the numerator and the denominator are polynomials
- Understand that a common denominator is a group of fractions that have a shared denominator
- Understand when adding and/or subtracting rational expressions with common denominators, combine like terms in the numerator (Simplify the new fraction if needed)

***MA.912.AR.2 Write, solve and graph linear equations, functions and inequalities in one and two variables.***

MA.912.AR.2.1

Given a real-world context, write and solve one-variable multi-step linear equations.

**Access Point**

MA.912.AR.2.AP.1 Given an equation in a real-world context, solve one-variable multi-step linear equations.

**Essential Understandings:**

- Understand the following vocabulary and symbols: add (+), subtract (-), multiply (x), divide ( $\div$ ), equal (=), linear equation, variable, like terms, coefficient, constant
- Understand how to add, subtract, multiply, and divide integers. (limited to 2-digit numbers)
- Understand combining like terms
- Understand to solve a one-variable multi-step linear equation, the variable must be isolated on one side
- Understand that all constants need to be on one

side of the equal sign

- Understand to get all constants on one side, add and subtract the same number to both sides of the equation to isolate the variable on one side and the constant on the other
- Understand if the coefficient is not one, multiply or divide both sides by the coefficient

$$2x + 4 = 10$$

$$\begin{array}{r} -4 \quad -4 \\ \hline 2x + 0 = 6 \end{array}$$

$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

- Use tools, (i.e., manipulatives, algebra tiles, software, equation calculators, etc.) to solve equations with one variable

MA.912.AR.2.2

Write a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context.

**Access Point**

MA.912.AR.2.AP.2 Select a linear two-variable equation to represent relationships between quantities from a graph, a written description or a table of values within a mathematical or real-world context.

**Essential Understandings:**

- Understand the following related vocabulary: y-intercept, slope, linear two-variable equation, graph, table, x-axis, y-axis, slope formula, positive slope, negative slope, vertical, horizontal.
- Understand that if the line is sloping upward from left to right, the slope of the line is positive.
- Understand that if the line is sloping downward from left to right, the slope of the line is negative.

	<ul style="list-style-type: none"> <li>• Understand that if the line is horizontal, the slope is 0.</li> <li>• Understand that if the line is vertical, the slope is undefined.</li> <li>• Understand the slope is the rise over the run.</li> <li>• Understand the y-intercept is where the line crosses the y-axis.</li> <li>• Understand to find the slope from a table, pick two points and put them in the slope formula.</li> <li>• Understand to find the y-intercept, locate the point where <math>x = 0</math>.</li> <li>• Understand the slope and the y-intercept will be used to create an equation (template, formula, etc.).</li> </ul>
MA.912.AR.2.3	<p>Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point.</p> <p><b>Access Point</b></p> <p>MA.912.AR.2.AP.3 Select a linear two-variable equation in slope intercept form for a line that is parallel or perpendicular to a given line and goes through a given point.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: y-intercept (<math>b</math>), slope (<math>m</math>), slope intercept form, linear two-variable equation, parallel, perpendicular, negative reciprocal, negative slope, positive slope, coefficient, vertical, horizontal, x-axis, y-axis.</li> <li>• Understand that if the line is sloping upward from left to right, the slope of the line is positive.</li> <li>• Understand that if the line is sloping downward from left to right, the slope of the line is negative.</li> <li>• Understand that if the line is horizontal, the slope is 0.</li> <li>• Understand that if the line is vertical, the slope</li> </ul>

is undefined.

- Understand the slope is the rise over the run.
- Understand the y-intercept is where the line crosses the y-axis.
- Understand which coefficient of the given linear two-variable equation is the slope.
- Understand that two parallel lines have the same slope.
- Understand that the slope of a line that is perpendicular to a given line is the negative reciprocal.

Ex.

$$\text{Slope: } m = \frac{2}{3}$$

$$\text{Negative reciprocal: } m = -\frac{3}{2}$$

$$\text{Slope: } m = 2$$

$$\text{Negative reciprocal: } -\frac{1}{2}$$

- Understand to create the equation of a line that is parallel to a given line, use the slope of the given line and a given point. (template, formulas, etc.).
- Understand to create the equation of a line that is perpendicular to a given line, use the negative reciprocal of the slope of the given line and a given point (template, formulas, etc.).

MA.912.AR.2.4

Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.

**Access Point**

MA.912.AR.2.AP.4 Given a table, equation or written description of a linear function, select a graph of that function and determine at least two key features (can include domain, range, y-intercept or slope).

**Essential Understandings:**

- Understand the following related vocabulary: x-axis, y-axis, domain, range, linear function, y-intercept (b), slope (m), graph, table, linear, variable, negative slope, positive slope, horizontal, vertical.
- Understand key features of a linear function

	<p>(can include domain, range, y-intercept, or slope).</p> <ul style="list-style-type: none"> <li>• Understand that if the slope is positive, the line on the graph rises upward from left to right.</li> <li>• Understand that if the slope is negative the line on the graph will fall downward from left to right.</li> <li>• Understand that if the slope is zero, the line on the graph is horizontal.</li> <li>• Understand that if the slope is undefined, the line on the graph is vertical.</li> <li>• Understand the slope is the rise over the run.</li> <li>• Understand the y-intercept is where the line crosses the y-axis.</li> <li>• Understand that the domain is all the x-values.</li> <li>• Understand that the range is all the y-values.</li> <li>• Understand that key features are used to create the graph.</li> </ul>
MA.912.AR.2.5	<p>Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine domain constraints in terms of the context.</p> <p><b>Access Point</b></p> <p>MA.912.AR.2.AP.5 Given a mathematical and/or real-world problem that is modeled with linear functions, solve the mathematical problem, or select the graph using key features (in terms of context) that represents this model.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: x-axis, y-axis, labels, scales, domain, linear function, y-intercept (b), slope (m), graph, add (+), subtract (-), multiply (x), divide (<math>\div</math>), equal (=), linear, variable</li> <li>• Understand key features of a linear function (can include domain, range, y-intercept, or slope).</li> <li>• Understand that if the slope is positive, the line on the graph rises upward from left to right.</li> <li>• Understand that if the slope is negative the line on the graph will fall downward from left to right</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that if the slope is zero, the line on the graph is horizontal.</li> <li>• Understand that if the slope is undefined, the line on the graph is vertical.</li> <li>• Understand the slope is the rise over the run.</li> <li>• Understand the y-intercept is where the line crosses the y-axis.</li> <li>• Understand that the domain is all the x-values.</li> <li>• Understand that the range is all the y-values.</li> <li>• Understand the slope (rate of change) and y-intercept (if the equation is in y-intercept form, <math>y = mx + b</math>, the constant (b) is where the line crosses the y-axis) from a real-world problem.</li> </ul>
MA.912.AR.2.6	<p>Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.</p> <p><b>Access Point</b></p> <p>MA.912.AR.2.AP.6 Given a mathematical and/or real-world context, select a one-variable linear inequality that represents the solution algebraically or graphically.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: number line, one-variable linear inequality, add (+), subtract (-), multiply (x), divide (<math>\div</math>), equal (=), Greater than (&gt;), Less than (&lt;), greater than or equal to (<math>\geq</math>), less than or equal to (<math>\leq</math>), variables, closed circle, open circle, positive direction, negative direction, like terms, coefficient, constant</li> <li>• Understand how to add, subtract, multiply, and divide integers. (limited to 2 digit numbers)</li> <li>• Understand combining like terms.</li> <li>• Understand to solve a one-variable multi-step linear inequalities, the variable must be isolated on one side.</li> <li>• Understand that all constants need to be on one side of the inequality.</li> <li>• Understand to get all constants on one side,</li> </ul>

	<p>add and subtract the same number to both sides of the inequality to isolate the variable on one side and the constant on the other.</p> <ul style="list-style-type: none"> <li>• Understand if the coefficient is not one, multiply or divide both sides by the coefficient.</li> <li>• Understand that a one-variable linear inequality can be represented on a number line.</li> <li>• Understand on a number line that when the equation or real-world context is <math>&gt;</math> or <math>&lt;</math>, the point is represented by an open circle.</li> <li>• Understand on a number line that when the equation or real-world context is <math>\geq</math> or <math>\leq</math>, the point is represented with a closed circle.</li> <li>• Understand that if the equation is <math>\geq</math> or <math>&gt;</math> then the graph goes in a positive direction (to the right).</li> <li>• Understand that if the equation is <math>\leq</math> or <math>&lt;</math> then the graph goes in a negative direction (to the left).</li> </ul>
MA.912.AR.2.7	<p>Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context.</p> <p><b>Access Point</b></p> <p>MA.912.AR.2.AP.7 Select a two-variable linear inequality to represent relationships between quantities from a graph.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: boundary line, two-variable linear inequality, slope (<math>m</math>), <math>y</math>-intercept (<math>b</math>), graph, shading a graph, add (+), subtract (-), multiply (<math>\times</math>), divide (<math>\div</math>), equal (=), Greater than (<math>&gt;</math>), Less than (<math>&lt;</math>), greater than or equal to (<math>\geq</math>), less than or equal to (<math>\leq</math>), variables, coordinate point, <math>x</math>-axis, <math>y</math>-axis, horizontal, vertical</li> <li>• Understand the slope (rise over run) and the <math>y</math>-intercept (where the line crosses the <math>y</math>-axis) of a two-variable linear inequality.</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that a dotted boundary line on a graph of a two-variable linear inequality represents less than (<math>&lt;</math>) or greater than (<math>&gt;</math>) .</li> <li>• Understand that a solid boundary line on a graph of a two-variable linear inequality represents less than or equal to (<math>\leq</math>) or greater than or equal to (<math>\geq</math>)</li> <li>• Identify above and below the boundary line.</li> <li>• Understand if the graph of a two-variable linear inequality is shaded above the boundary line, the graph represents greater than or greater than or equal to.</li> <li>• Understand if the graph of a two-variable linear inequality is shaded below the boundary line, the graph represents less than or less than or equal to.</li> </ul>
MA.912.AR.2.8	<p>Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.</p> <p><b>Access Point</b>  MA.912.AR.2.AP.8 Given a two-variable linear inequality, select a graph that represents the solution.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: boundary line, two-variable linear inequality, slope (<math>m</math>), <math>y</math>-intercept (<math>b</math>), graph, shading a graph, add (<math>+</math>), subtract (<math>-</math>), multiply (<math>\times</math>), divide (<math>\div</math>), equal (<math>=</math>), Greater than (<math>&gt;</math>), Less than (<math>&lt;</math>), greater than or equal to (<math>\geq</math>), less than or equal to (<math>\leq</math>), variables, coordinate point, <math>x</math>-axis, <math>y</math>-axis, horizontal, vertical</li> <li>• Understand the slope (rise over run) and the <math>y</math>-intercept (where the line crosses the <math>y</math>-axis, or <math>x = 0</math>) of a two-variable linear inequality</li> <li>• Understand that two-variable linear inequality is in the form of one of the following:  Less than: <math>y &lt; mx + b</math>  Less than or equal to: <math>y \leq mx + b</math>  Greater than: <math>y &gt; mx + b</math>  Greater than or equal to: <math>y \geq mx + b</math></li> <li>• Understand that a dotted boundary line on a graph of a two-variable linear inequality</li> </ul>



	<p>represents less than (<math>&lt;</math>) or greater than (<math>&gt;</math>).</p> <ul style="list-style-type: none"> <li>• Understand that a solid boundary line on a graph of a two-variable linear inequality represents less than or equal to or greater than or equal to</li> <li>• Identify above and below the boundary line.</li> <li>• Understand if the graph of a two-variable linear inequality is shaded above the boundary line, the graph represents greater than or greater than or equal to.</li> <li>• Understand if the graph of a two-variable linear inequality is shaded below the boundary line, the graph represents less than or less than or equal to.</li> <li>• Understand that a linear inequality divides the coordinate plane into two parts by a boundary line where one represents the solutions of the inequality (Any coordinate point that falls in the shaded region or on the boundary line if it is solid line, is the solution.)</li> </ul>
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***MA.912.AR.3 Write, solve and graph quadratic equations, functions and inequalities in one and two variables.***

MA.912.AR.3.1	Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.
	<p><b>Access Point</b></p> <p>MA.912.AR.3.AP.1 Given a one-variable quadratic equation from a mathematical or real-world context, select the solution to the equation over the real number system.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: add (+), subtract (-), multiply (x), divide (<math>\div</math>), equal (=), one-variable, quadratic expression, quadratic equation, quadratic formula, real number system, factors, factored form, coefficient, exponent</li> <li>• Understand the factors of real numbers</li> <li>• Understand to determine the solutions to quadratic equations use factoring tools/methods (E.g., Algebra tiles, guess and</li> </ul>

	<p>check, quadratic formula, online tools, etc.)</p> <ul style="list-style-type: none"> <li>• Understand the solution to a quadratic equation is what numerical value is substituted for the variable to make the equation equal to zero</li> </ul>
<p>MA.912.AR.3.2</p>	<p>Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real and complex number systems.</p> <p><b>Access Point</b></p> <p>MA.912.AR.3.AP.2 Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: add (+), subtract (-), multiply (x), divide (<math>\div</math>), equal (=), one-variable, quadratic expression, quadratic equation, quadratic formula, real number system, factors, factored form, coefficient, exponent</li> <li>• Understand the factors of real numbers.</li> <li>• Understand to determine the solutions to quadratic equations use factoring tools/methods (E.g., Algebra tiles, guess and check, quadratic formula, online tools, etc.)</li> <li>• Understand the solution to a quadratic equation is what numerical value is substituted for the variable to make the equation equal to zero.</li> </ul>
<p>MA.912.AR.3.3</p>	<p><b>Given a mathematical or real-world context, write and solve one-variable quadratic inequalities over the real number system. Represent solutions algebraically or graphically.</b></p> <p><b>Access Point</b></p> <p>MA.912.AR.3.AP.3 Given a mathematical or real-world context, select a one-variable quadratic inequality over the real number system that represents the solution algebraically or graphically.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: add (+), subtract (-), multiply (x), divide</li> </ul>

( $\div$ ), equal (=), Greater than (>), less than (<), greater than or equal to ( $\geq$ ), less than or equal to ( $\leq$ ), one-variable, interval, quadratic expression, quadratic inequality, quadratic formula, real number system, factors, factored form, coefficient, exponent

- Understand the factors of real numbers.

Ex. 24

Factors: (2)(12); (3)(8);(4)(6);(1)(24)

Ex. 6

Factors: (1)(6); (2)(3)

- Understand the solutions to quadratic inequalities by using factoring tools/methods.

(E.g., Algebra tiles, guess and check, quadratic formula, online tools, etc.)

Ex.

Guess and Check: (provides the factors)

$x^2 + 7x + 6$  (Quadratic expression)

Find the Factors of the last term (6)

Factors: (1)(6); (2)(3)

If the last term is positive, you are adding to find the middle term. Choose the factors 6 and 1 because when added together they equal 7. The middle term is  $7x$ .

$$x^2 + 7x + 6$$

$$(x + 6)(x + 1) \text{ (Factored form)}$$

$$x^2 + 7x + 6 < 0 \text{ (quadratic inequality)}$$

$$(x + 6)(x + 1) < 0 \text{ (Factored form)}$$

Set each factor equal to zero

$$x + 6 = 0 \text{ or } x + 1 = 0$$

$$x = -6 \text{ or } x = -1 \text{ (zeros)}$$

MA.912.AR.3.4

Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

**Access Point**

MA.912.AR.3.AP.4 Select a quadratic function to represent the relationship between two quantities from a graph.

### Essential Understandings:

- Understand the following related vocabulary: two-variable, vertex, vertex form, quadratic function, graph, point on the graph, opens upward, opens downward, parabola, leading coefficient, positive, negative.
- Understand that the graph of a quadratic function is a parabola.
- Understand where the vertex is located on the graph. Ex: The highest point if the graph is open downward and the lowest point if the graph is open upward.
- Understand what the variables in the vertex form represent. Ex:  $y = a(x - h)^2 + k$ 
  - Vertex =  $(h, k)$  ( $h$  is the  $x$ -value,  $k$  is the  $y$ -value)
  - Leading coefficient =  $a$
  - Point on a graph =  $(x, y)$
- Understand when  $a$  is positive, the graph of the parabola opens upward.
- Understand when  $a$  is negative, the graph of the parabola opens downward.
- Understand that in the vertex form,  $h$  is replaced with the  $x$ -value of the vertex.
  - Ex. Vertex =  $(3, -1)$
  - $y = a(x - 3)^2 + k$
- Understand that in the vertex form,  $k$  is replaced with the  $y$ -value of the vertex.
  - Ex. Vertex =  $(3, -1)$
  - $y = a(x - 3)^2 - 1$
- Understand that in the vertex form, we will replace  $x$  and  $y$  with a point on the graph to find the variable  $a$
- Ex. Point on a graph  $(1, 7)$

$$7 = a(1 - 3)^2 - 1$$

$$7 = a(-2)^2 - 1$$

$$7 = 4a - 1$$

$$8 = 4a$$

$$2 = a$$

$$y = 2(x - 3)^2 - 1 \text{ (quadratic in vertex form)}$$

<p>MA.912.AR.3.5</p>	<p>Given the <math>x</math>-intercepts and another point on the graph of a quadratic function, write the equation for the function</p> <p><b>Access Point</b></p> <p>MA.912.AR.3.AP.5 Given the <math>x</math>-intercepts and another point on the graph of a quadratic function, select the equation for the function.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: quadratic function, graph, point on the graph, parabola, leading coefficient, <math>x</math>-intercepts, factored form, zeros, <math>x</math>-axis, <math>y</math>-axis.</li> <li>• Understand the factors of real numbers.</li> <li>• Understand that the <math>x</math>-intercepts are factors of the quadratic.</li> <li>• Understand that a quadratic in factored form is modeled by: <ul style="list-style-type: none"> <li>• <math>y = a(x - r_1)(x - r_2)</math></li> <li>• (with zeros at <math>r_1</math> and <math>r_2</math>)</li> </ul> </li> <li>• Understand that a point on the graph must be plugged in to solve for the leading coefficient which is <math>a</math>.</li> </ul>
<p>MA.912.AR.3.6</p>	<p>Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.</p> <p><b>Access Point</b></p> <p>MA.912.AR.3.AP.6 Given an expression or equation representing a quadratic function in vertex form, determine the vertex and zeros.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: vertex, vertex form, quadratic function, zeros, quadratic expression, quadratic equation.</li> <li>• Understand that the vertex form is <math>y = a(x - h)^2 + k</math>.</li> <li>• Understand that the vertex is <math>(h, k)</math>.</li> <li>• Understand when the equation is in vertex form, set the equation equal to zero and</li> </ul>

	solve for x to find the zeros.
MA.912.AR.3.7	<p>Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.</p> <p><b>Access Point</b>  MA.912.AR.3.AP.7 Given a table, equation or written description of a quadratic function, select the graph that represents the function.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: vertex, vertex form, quadratic function, graph, point on the graph, opens upward, opens downward, parabola, table, positive number, negative number, maximum point, minimum point, x-axis, y-axis.</li> <li>• Understand that the graph of a quadratic function is a parabola.</li> <li>• Understand that the vertex form of a quadratic is <math>y = a(x - h)^2 + k</math></li> <li>• Understand what makes the graph open upward or downward (parabola opens upward when a is positive and parabola opens downward when a is negative)</li> <li>• Understand that the vertex is the minimum or maximum point on the graph of the parabola.</li> <li>• Understand when given an equation in vertex form, the vertex is <math>(h, k)</math></li> <li>• Understand that an additional point (s) will need to be found on the parabola to identify which graph represents the equation.</li> <li>• Understand when given a table of x and y-values, place the points on a coordinate graph and connect the points to create a parabola.</li> <li>• Understand that a written description can be given to describe the graph of a parabola</li> </ul>
MA.912.AR.3.8	Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the

	<p>context.</p> <p><b>Access Point</b></p> <p>MA.912.AR.3.AP.8 Given a mathematical and/or real-world problem that is modeled with quadratic functions, solve the mathematical problem, or select the graph using key features (in terms of context) that represents this model</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: vertex, vertex form, standard form, quadratic function, graph, opens upward, opens downward, parabola, positive number, negative number, maximum point, minimum point, x-axis, y-axis, x-intercept, y-intercept, axis of symmetry.</li> <li>• Understand that the graph of a quadratic function is a parabola.</li> <li>• Understand that the vertex form of a quadratic is <math>y = a(x - h)^2 + k</math></li> <li>• Understand that the standard form of a quadratic is <math>y = ax^2 + bx + c</math></li> <li>• Understand what makes the graph open upward or downward. (parabola opens upward when a is positive and parabola opens downward when a is negative)</li> <li>• Understand that the vertex is the minimum or maximum point on the graph of the parabola.</li> <li>• Understand in a real-world problem, the vertex represents maximum profit, maximum height, minimum cost for production, etc.</li> <li>• Understand when given an equation in vertex form, the vertex is <math>(h, k)</math></li> <li>• Understand when given an equation in standard form, the vertex is found by <math>\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)</math></li> <li>• Understand that key features may include vertex, axis of symmetry, x-intercept(s), and y-intercept(s)</li> </ul>
MA.912.AR.3.9	Given a mathematical or real-world context, write two-

variable quadratic inequalities to represent relationships between quantities from a graph or a written description.

**Access Point**

MA.912.AR.3.AP.9 Select two-variable quadratic inequalities to represent relationships between quantities from a graph or a written description.

**Essential Understandings:**

- Understand the following related vocabulary: vertex, vertex form, standard form, quadratic function, graph, opens upward, opens downward, parabola, positive number, negative number, maximum point, minimum point, x-axis, y-axis, x-intercept, y-intercept, axis of symmetry, quadratic inequality, boundary line, shaded, key features, Greater than ( $>$ ), Less than ( $<$ ), greater than or equal to ( $\geq$ ), less than or equal to ( $\leq$ ).
- Understand that the graph of a quadratic function is a parabola.
- Understand that key features may include vertex, axis of symmetry, x-intercept(s), and y-intercept(s)
- Understand to create the inequality that represents the quadratic function more than one key feature will be needed.

Ex. vertex, x-intercept, y-intercept, etc.

- Understand that the vertex is the minimum or maximum point on the graph of the parabola.
- Understand when given an equation in vertex form, the vertex is  $(h, k)$ .
- Understand when given an equation in standard form, the vertex is found by  $(\frac{-b}{2a}, f(\frac{-b}{2a}))$
- Understand what makes the graph open upward or downward. (Parabola opens upward when a is positive and parabola opens downward when a is negative).
- Understand that the vertex form of a quadratic is  $y = a(x - h)^2 + k$ .



	<ul style="list-style-type: none"> <li>• Understand that the standard form of a quadratic is <math>y = ax^2 + bx + c</math>.</li> <li>• Understand if the inequality includes <math>&lt;</math> or <math>&gt;</math>, the boundary lines of the parabola will be dashed.</li> <li>• Understand if the inequality includes <math>\leq</math> or <math>\geq</math>, the boundary lines of the parabola will be solid.</li> <li>• Understand with quadratic inequalities:         If the inequality is <math>&lt;</math>, shade below the dashed boundary line.        If the inequality is <math>&gt;</math>, shade above the dashed boundary line.        If the inequality is <math>\leq</math>, shade below the solid boundary line .        If the inequality is <math>\geq</math>, shade above the solid boundary line.</li> <li>• Understand in a real-world problem, the vertex represents maximum profit, maximum height, minimum cost for production, etc.</li> </ul>
<p>MA.912.AR.3.1 0</p>	<p>Given a mathematical or real-world context, graph the solution set to a two-variable quadratic inequality.</p> <p><b>Access Point</b>        MA.912.AR.3.AP.10 Given a mathematical or real-world context, graph the solution set to a two-variable quadratic inequality.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: vertex, vertex form, standard form, quadratic function, graph, opens upward, opens downward, parabola, positive number, negative number, maximum point, minimum point, x-axis, y-axis, x-intercept, y-intercept, axis of symmetry, quadratic inequality, boundary line, shaded, key features, Greater than (<math>&gt;</math>), Less than (<math>&lt;</math>), greater than or equal to (<math>\geq</math>), less than or equal to (<math>\leq</math>).</li> <li>• Understand that the graph of a quadratic function is a parabola.</li> <li>• Understand that key features may include</li> </ul>

vertex, axis of symmetry, x-intercept(s), and y-intercept(s)

- Understand that the vertex is the minimum or maximum point on the graph of the parabola.
- Understand when given an equation in vertex form, the vertex is  $(h, k)$
- Understand when given an equation in standard form, the vertex is found by  $(\frac{-b}{2a}, f(\frac{-b}{2a}))$
- Understand what makes the graph open upward or downward (Parabola opens upward when  $a$  is positive and parabola opens downward when  $a$  is negative).
- Understand that the vertex form of a quadratic is  $y = a(x - h)^2 + k$ .
- Understand that the standard form of a quadratic is  $y = ax^2 + bx + c$
- Understand if the inequality includes  $<$  or  $>$ , the boundary lines of the parabola will be dashed.
- Understand if the inequality includes  $\leq$  or  $\geq$ , the boundary lines of the parabola will be solid.
- Understand with quadratic inequalities:

If the inequality is  $<$ , shade below the dashed boundary line.

If the inequality is  $>$ , shade above the dashed boundary line.

If the inequality is  $\leq$ , shade below the solid boundary line.

If the inequality is  $\geq$ , shade above the solid boundary line.

***MA.912.AR.4 Write, solve and graph absolute value equations, functions and inequalities in one and two variables.***

MA.912.AR.4.1

Given a mathematical or real-world context, write and solve one-variable absolute value equations.

**Access Point**

MA.912.AR.4.AP.1 Solve a one variable absolute value equation.

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: add (+), subtract (-), multiply (x), divide (<math>\div</math>), equal (=), absolute value, negative number, positive number, distance, integer, two step equation, variable</li> <li>• Understand how to add, subtract, multiply and divide integers <ul style="list-style-type: none"> <li>• Understand how to solve two step equations</li> <li>• Understand that the absolute value represents the distance a number is from zero</li> <li>• Understand that distance is always a positive number or zero (distance from 0 to -3 is 3 and the distance from 0 to 3 is 3)</li> <li>• Understand to solve the absolute value equation, solve for a negative and a positive value (there are two numbers that are the same distance to zero, exception, the number zero)</li> </ul> </li> </ul>
MA.912.AR.4.2	<p>Given a mathematical or real-world context, write and solve one-variable absolute value inequalities. Represent solutions algebraically or graphically.</p> <p><b>Access Point</b></p> <p>MA.912.AR.4.AP.2 Solve a one-variable absolute value inequality. Represent solutions algebraically or graphically.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: inequality, absolute value, negative number, positive number, distance, graph, greater than (&gt;), less than (&lt;), greater than or equal to (<math>\geq</math>), less than or equal to (<math>\leq</math>)</li> <li>• Understand that the absolute value represents the distance a number is from zero</li> <li>• Understand that an inequality of less than (&lt;) or less than or equal to (<math>\leq</math>) for the absolute value of x can be represented as <math>-a &lt; x &lt; a</math> or <math>-a \leq x \leq a</math></li> </ul>

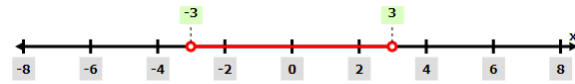
Ex

$$|x| < a$$

$$-a < x < a$$

$$|x| < 3$$

$$-3 < x < 3$$



- Understand that an inequality of greater than ( $>$ ) or greater than or equal to ( $\geq$ ) for the absolute value of  $x$  can be represented as  $x < -a$  or  $x > a$  (for greater than) and  $x \leq -a$  or  $x \geq a$  (for greater than or equal to)

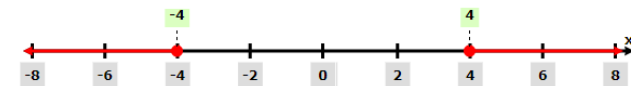
Ex.

$$|x| \geq a$$

$$x \leq -a \text{ or } x \geq a$$

$$|x| \geq 4$$

$$x \leq -4 \text{ or } x \geq 4$$



MA.912.AR.4.3

Given a table, equation or written description of an absolute value function, graph that function and determine its key features.

**Access Point**

MA.912.AR.4.AP.3 Given a table, equation or written description of an absolute value function, select the graph that represents the function.

**Essential Understandings:**

- Understand the following related vocabulary:

absolute value, vertex, negative number, positive number, interior of absolute value function, table, maximum point, minimum point, ordered pairs, add (+), subtract (-), multiply ( $\times$ ), divide ( $\div$ ), equal (=), integer, two step equation, x-axis, y-axis, variable, standard form

- Understand how to add, subtract, multiply and divide integers.
- Understand how to solve two step equations.
- Understand that the graph of a two variable absolute value function is in the shape of a V.
- Understand that the standard form of an absolute value is  $y = a|x - h| + k$ .
- Understand that the vertex is  $(h, k)$ .
- Understand that the vertex is the maximum or minimum point on the absolute value graph.
- Understand that when  $a$  is positive the graph opens upward and when  $a$  is negative, the graph opens downward.
- Understand that to find the x-value of the vertex of an absolute value function, set the interior  $(x - h)$  of the absolute value equal to zero and solve for  $x$ .
- Understand that the y-value of the vertex of an absolute value function is  $k$ .
- Understand when graphing an absolute value graph, graph the vertex point first.
- Understand after graphing the vertex, create a table of ordered pairs using values on the left and right side of the vertex.

Ex. (vertex =  $(0,0)$ )

x	y =  x
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

- Understand that the ordered pairs in the table create an absolute value graph

MA.912.AR.4.4

Solve and graph mathematical and real-world problems that are modeled with absolute value functions. Interpret key features and determine constraints in terms of the context.

**Access Point**

MA.912.AR.4.AP.4 Given a mathematical and/or real-world problem that is modeled with absolute value functions, solve the mathematical problem, or select the graph using key features (in terms of context) that represents this model.

**Essential Understandings:**

- Understand the following related vocabulary: absolute value, vertex, negative number, positive number, absolute value function, interior of absolute value function, maximum point, minimum point, ordered pairs, domain, range.
- Understand that the vertex is the maximum or minimum point on the absolute value graph.
- Understand what makes the graph open upward or downward.

Ex.

$$y = a|x - h| + k$$

$$y = 2|x|$$

$$a = 2$$

Absolute value function opens upward when  $a$  is a positive number.

$$y = -3|x|$$
$$a = -3$$

Absolute value function opens downward when  $a$  is a negative number.

- Understand that to find the  $x$ -value of the vertex of an absolute value function, set the interior of the absolute value equal to zero.

Ex.

$$y = |x|$$
$$x = 0$$

Ex.

$$y = |x + 2|$$
$$x + 2 = 0$$
$$x = -2$$

- Understand that the  $y$ -value of the vertex of an absolute value function is  $k$ .

Ex.

$$y = a|x - h| + k$$
$$y = |x|$$
$$k = 0$$

Vertex (0, 0)

Ex.

$$y = |x| + 3$$
$$k = 3$$

Vertex (0, 3)

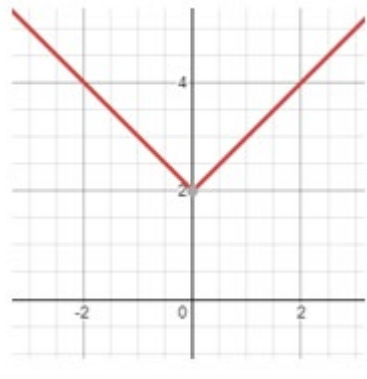
- Understand that the value of  $k$  moves the graph up or down.

Ex.

$$y = |x| + 2$$
$$k = 2$$

Because  $k$  is 2, the graph shifts two places up

$$f(x) = |x| + 2$$



- Understand that the value of  $h$  moves the graph left or right.

Ex

$$y = |x + 3|$$

Find the zero of the interior of the absolute value.

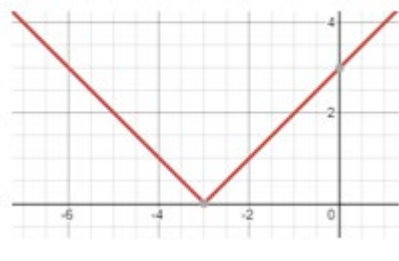
$$x + 3 = 0$$

$$x = -3$$

Therefore  $h = -3$

Therefore the graph has shifted to the left three places.

$$f(x) = |x + 3|$$



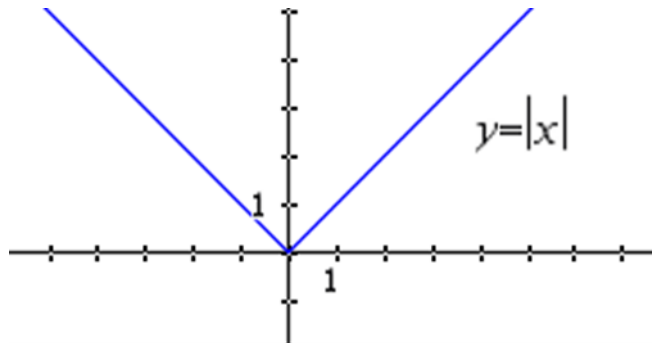
- Understand when graphing an absolute value graph, graph the vertex point first.
- Understand after graphing the vertex, create a table of ordered pairs on the left and right



side of the vertex.

x	y =  x
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

- Understand that the ordered pairs in the table create an absolute value graph .



- Understand that the domain is all the possible x-values for a graph.
- Understand that the range is all the possible y-values for a graph.

**MA.912.AR.5 Write, solve and graph exponential and logarithmic equations and functions in one and two variables.**

MA.912.AR.5.2

Solve one-variable equations involving logarithms or exponential expressions. Interpret solutions as viable in terms of the context and identify any extraneous solutions.

**Access Point**

MA.912.AR.5.AP.2 Solve one-variable equations involving logarithms or exponential expressions. Identify any extraneous solutions.

**Essential Understandings:**

- Understand the concepts and vocabulary:

algebraic logarithmic expression, exponent, equivalent, logarithm, base, properties, extraneous solution, simplified, undefined.

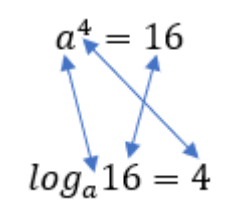
- Understand how to identify expressions with exponents.

E.g.,  $(a^4)$

- Understand what the exponent represents in expanded form.

E.g.,  $a^4 = a \times a \times a \times a$

- Understand what question a logarithm asks.



- Understand the rules for logarithms.
- Understand that the rules for logarithms will need to be applied to simplify expressions to find the solutions.
- Understand the following would result in an extraneous solution:  $\log(\text{negative number})$  and  $\log 0$ . Both solutions are undefined.

MA.912.AR.5.3

Given a mathematical or real-world context, classify an exponential function as representing growth or decay.

**Access Point**

MA.912.AR.5.AP.3 Given a real-world context, identify an exponential function as representing growth or decay.

**Essential Understandings:**

- Understand the following terms and vocabulary:  $x$ -axis,  $y$ -axis, increase, decrease, left, right, growth, decay, exponential function, exponential.
- Understand that an exponential function that represents growth will quickly increase from left to right.

	<ul style="list-style-type: none"> <li>• Understand that an exponential function that represents decay will quickly decrease from left to right.</li> <li>• Understand that growth can be represented by a pandemic, rabbits, mice, fleas, population, etc.</li> <li>• Understand that decay can be represented by radioactive materials, population, something that cools (coffee, soup), etc.</li> </ul>
MA.912.AR.5.4	<p>Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.</p> <p><b>Access Point</b></p> <p>MA.912.AR.5.AP.4 Select an exponential function to represent two quantities from a graph or a table of values.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: <i>x</i>-axis, <i>y</i>-axis, <i>x</i>-value, <i>y</i>-value, left, right, increase, exponential function, exponential, table, graph, constant, common ratio, initial value, definable point, consecutive.</li> <li>• Understand when given a table of an exponential function the <i>x</i>-values will increase by a constant value and the <i>y</i>-values will increase by a common ratio.</li> <li>• Understand when given the exponential equation <math>y = ab^x</math> the variable <i>a</i> represents the initial value and the variable <i>b</i> represents the ratio between the <i>y</i>-values (<math>a \neq 0, b \neq 1, \text{ and } b &gt; 0</math>).</li> <li>• Understand when a graph of the exponential function crosses the <i>y</i>-axis at a definable point the <i>y</i>-intercept is the initial value.</li> <li>• Understand when given a graph, to calculate the value for the variable <i>b</i> select two consecutive definable points and calculate the ratio between the <i>y</i>-values.</li> </ul>

<p>MA.912.AR.5.5</p>	<p>Given an expression or equation representing an exponential function, reveal the constant percent rate of change per unit interval using the properties of exponents. Interpret the constant percent rate of change in terms of a real-world context.</p> <p><b>Access Point</b></p> <p>MA.912.AR.5.AP.5 Given an expression or equation representing an exponential function, reveal the constant percent rate of change per unit interval using the properties of exponents.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: algebraic expression, exponents, variable, base number, integers, growth, decay, constant percent change, initial value, properties of exponents.</li> <li>• Understand how to identify the parts of an algebraic expression.</li> </ul> <p>E.g., <math>x^7</math> where <math>x</math> is the base number and 7 is the exponent.</p> <ul style="list-style-type: none"> <li>• Understand how to identify expressions with exponents.</li> </ul> <p>E.g., <math>(x^4)(x^3)</math></p> <ul style="list-style-type: none"> <li>• Understand and use the properties of exponents to simplify algebraic expressions.</li> <li>• Understand the following formula: <math>f(x) = ab^x</math> (a = initial value, b = the growth or decay factor, x = constant percentage change).</li> </ul>
<p>MA.912.AR.5.6</p>	<p>Given a table, equation or written description of an exponential function, graph that function and determine its key features.</p> <p><b>Access Point</b></p> <p>MA.912.AR.5.AP.6 Given a table, equation or written description of an exponential function, select the graph that represents the function.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and</li> </ul>

vocabulary: x-axis, y-axis, x-value, y-value, left, right, increase, decrease, growth, decay, exponential function, exponential, table, standard form, graph, constant, common ratio, initial value, definable point, consecutive.

- Understand that an exponential function that represents growth will quickly increase from left to right.
- Understand that an exponential function that represents decay will quickly decrease from left to right.
- Understand that growth can be represented by a pandemic, rabbits, mice, fleas, population, etc.
- Understand that decay can be represented by radioactive materials, population, something that cools (coffee, soup), etc.
- Understand when given a table of an exponential function the x-values will increase by a constant value and the y-values will increase by a common ratio.
- Understand when given the exponential equation  $y = ab^x$  the variable  $a$  represents the initial value and the variable  $b$  represents the ratio between the y-values ( $a \neq 0, b \neq 1, \text{ and } b > 0$ ).
- Understand when a graph of the exponential function crosses the y-axis at a definable point the y-intercept is the initial value variable  $a$
- Understand when given a graph, to calculate the value for the variable  $b$  select two consecutive definable points and calculate the ratio between the y-values.
- Understand that the standard form of an exponential function that represents growth is  $y = a(1 + r)^x$  where  $a$  is the initial value ( $a > 0$ ),  $r$  is the rate of growth ( $r > 0$ ),  $x$  is time.
- Understand for exponential growth, as  $x$  increases,  $y$  grows exponentially.

	<ul style="list-style-type: none"> <li>• Understand that the standard form of an exponential function that represents decay is <math>y = a(1 - r)^x</math> where <math>a</math> is the initial value (<math>a &gt; 0</math>), <math>r</math> is the rate of decay (<math>0 &lt; r &lt; 1</math>), <math>x</math> is time.</li> <li>• Understand for exponential decay, as <math>x</math> increases, <math>y</math> decreases exponentially</li> </ul>
MA.912.AR.5.7	<p>Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context.</p> <p><b>Access Point</b></p> <p>MA.912.AR.5.AP.7 Given a mathematical and/or real-world problem that is modeled with exponential functions, solve the mathematical problem, or select the graph using key features (in terms of context) that represents this model.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: <math>x</math>-axis, <math>y</math>-axis, <math>x</math>-value, <math>y</math>-value, left, right, increase, decrease, growth, decay, exponential function, exponential, standard form, graph, constant, common ratio, initial value, definable point, properties of exponents.</li> <li>• Understand and use the properties of exponents to simplify algebraic expressions.</li> <li>• Understand when given the exponential equation <math>y = ab^x</math> the variable <math>a</math> represents the initial value and the variable <math>b</math> represents the ratio between the <math>y</math>-values (<math>a \neq 0, b \neq 1, \text{ and } b &gt; 0</math>).</li> <li>• Understand that an exponential function that represents growth will quickly increase from left to right.</li> <li>• Understand that an exponential function that represents decay will quickly decrease from left to right.</li> <li>• Understand that growth can be represented by a pandemic, rabbits, mice, fleas, population, etc.</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that decay can be represented by radioactive materials, population, something that cools (coffee, soup), etc.</li> <li>• Understand when a graph of the exponential function crosses the y-axis at a definable point the y-intercept is the initial value variable <math>a</math>.</li> <li>• Understand that the standard form of an exponential function that represents growth is <math>y = a(1 + r)^x</math> where <math>a</math> is the initial value (<math>a &gt; 0</math>), <math>r</math> is the rate of growth (<math>r &gt; 0</math>), <math>x</math> is time.</li> <li>• Understand for exponential growth, as <math>x</math> increases, <math>y</math> grows exponentially.</li> <li>• Understand that the standard form of an exponential function that represents decay is <math>y = a(1 - r)^x</math> where <math>a</math> is the initial value (<math>a &gt; 0</math>), <math>r</math> is the rate of decay (<math>0 &lt; r &lt; 1</math>), <math>x</math> is time.</li> <li>• Understand for exponential decay, as <math>x</math> increases, <math>y</math> decreases exponentially.</li> </ul>
MA.912.AR.5.8	<p>Given a table, equation or written description of a logarithmic function, graph that function and determine its key features.</p> <p><b>Access Point</b></p> <p>MA.912.AR.5.AP.8 Given an equation of a logarithmic function, select the graph of that function.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: parent function, logarithmic function, base number, domain, increase, decrease, positive real numbers, range, asymptote, graph, stretch, shrink, shift, horizontal, vertical, reflect, x-axis, y-axis.</li> <li>• Understand the parent function for a logarithmic function is <math>y = \log_b x</math></li> <li>• Understand the key features for the parent function: The graph of the function crosses the x-axis at (1,0)</li> <li>• The base number is <math>b</math> (if <math>b &gt; 1</math>, the graph</li> </ul>

	<p>increases, if <math>0 &lt; b &lt; 1</math>, the graph decreases)</p> <p>The domain is all positive real numbers (not including zero)  The range is all real numbers.  The graph has an asymptote at the <math>y</math>-axis.</p> <ul style="list-style-type: none"> <li>• Understand the following formula for log transformations. <math>y = a\log_b(x - h) + k</math></li> </ul> <p>If <math>a &lt; 0</math>, the graph reflects over the <math>x</math>-axis .  If <math> a  &gt; 1</math>, the graph stretches  If <math>0 &lt;  a  &lt; 1</math>, the graph shrinks  <math>h</math> shifts the graph horizontally right and left  <math>k</math> shifts the graph vertically up and down</p>
<p>MA.912.AR.5.9</p>	<p>Solve and graph mathematical and real-world problems that are modeled with logarithmic functions. Interpret key features and determine constraints in terms of the context.</p> <p><b>Access Point</b></p> <p>MA.912.AR.5.AP.9 Given a mathematical and/or real-world problem that is modeled with logarithmic functions, solve the mathematical problem, or select the graph using key features (in terms of context) that represents this model.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: parent function, logarithmic function, base number, domain, increase, decrease, positive real numbers, range, asymptote, graph, stretch, shrink, shift, horizontal, vertical, reflect, <math>x</math>-axis, <math>y</math>-axis, extraneous solution, rules for logarithms.</li> <li>• Understand the rules for logarithms.</li> <li>• Understand that the rules for logarithms will need to be applied to simplify expressions to find the solutions.</li> <li>• Understand the following would result in an extraneous solution: <math>\log(\textit{negative number})</math> and <math>\log 0</math></li> </ul> <p>Both solutions are undefined</p>



- Understand the parent function for a logarithmic function is  $y = \log_b x$
- Understand the key features for the parent function:

The graph of the function crosses the  $x$ -axis at  $(1,0)$

The base number is  $b$  (if  $b > 1$ , the graph increases, if  $0 < b < 1$ , the graph decreases)

The domain is all positive real numbers (not including zero)

The range is all real numbers.

The graph has an asymptote at the  $y$ -axis.

- Understand the following formula for log transformations.  $y = a \log_b(x - h) + k$

If  $a < 0$ , the graph reflects over the  $x$ -axis

If  $|a| > 1$ , the graph stretches

If  $0 < |a| < 1$ , the graph shrinks

$h$  shifts the graph horizontally right and left

$k$  shifts the graph vertically up and down

***MA.912.AR.6 Solve and graph polynomial equations and functions in one and two variables.***

MA.912.AR.6.1

Given a mathematical or real-world context, when suitable factorization is possible, solve one-variable polynomial equations of degree 3 or higher over the real and complex number systems.

**Access Point**

MA.912.AR.6.AP.1 Solve one-variable polynomial equations of degree 3 or higher in factored form, over the real number system.

**Essential Understandings:**

- Understand the following terms and vocabulary: one-variable polynomial equations, factored form, real number system, degree 3, exponent, solution, zeros.
- Understand a degree 3 of a one-variable polynomial equation could have three solutions, a degree 4 of a one-variable polynomial equation could have four solutions, etc. (a degree 3 means the largest exponent is a 3)

	<ul style="list-style-type: none"> <li>Understand the following:   <math>f(x) = (x-a)(x-b)(x-c)</math>, <math>a</math>, <math>b</math>, and <math>c</math> are the zeros or solutions to the polynomial</li> </ul>
MA.912.AR.6.5	<p>Sketch a rough graph of a polynomial function of degree 3 or higher using zeros, multiplicity and knowledge of end behavior.</p> <p><b>Access Point</b></p> <p>MA.912.AR.6.AP.5 Create a rough graph of a polynomial function of degree 3 or higher (in factored form) using zeros, multiplicity and knowledge of end behavior.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand the following terms and vocabulary: polynomial equations, factored form, real number system, even, odd, zeros, crosses, graph, x-axis, y-axis, end behavior, solutions, exponent, leading coefficient, <math>\infty</math>, <math>\rightarrow</math>, <math>-\infty</math>, positive, negative.</li> <li>Understand a degree 3 of a polynomial equation could have three solutions, a degree 4 of a polynomial equation could have four solutions, etc. (a degree 3 means the largest exponent is a 3)</li> <li>Understand the following: <math>f(x) = (x-a)(x-b)(x-c)</math>, <math>a</math>, <math>b</math>, and <math>c</math> are the zeros or solutions to the polynomial.</li> <li>Understand the following: <math>f(x) = (x-r)^n</math>         If <math>n</math> is even, the graph touches the x-axis at <math>r</math>        If <math>n</math> is odd, the graph crosses the x-axis at <math>r</math></li> <li>Understand the following end behavior:         If the degree is even, and the leading coefficient is positive as, <math>x \rightarrow \infty, f(x) \rightarrow \infty</math> and <math>x \rightarrow -\infty, f(x) \rightarrow \infty</math>        If the degree is even, and the leading coefficient is negative as, <math>x \rightarrow \infty, f(x) \rightarrow -\infty</math> and <math>x \rightarrow -\infty, f(x) \rightarrow -\infty</math>        If the degree is odd, and the leading coefficient is positive as, <math>x \rightarrow \infty, f(x) \rightarrow \infty</math> and <math>x \rightarrow -\infty, f(x) \rightarrow -\infty</math></li> </ul>

If the degree is odd, and the leading coefficient is negative as,  $x \rightarrow \infty, f(x) \rightarrow -\infty$  and  $x \rightarrow -\infty, f(x) \rightarrow \infty$

**MA.912.AR.9 Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.**

MA.912.AR.9.1

Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.

**Access Point**

MA.912.AR.9.AP.1

Given an algebraic or graphical system of two-variable linear equations, select the solution to the system of equations.

**Essential Understandings:**

- Understand the following terms and vocabulary: linear equation, solution to a system of linear equations, one solution, infinitely many solutions, no solutions, add (+), subtract (-), multiply (x), divide ( $\div$ ), equal (=), integer, two step equation, x-axis, y-axis, variable, ordered pair
- Understand how to add, subtract, multiply and divide integers
- Understand how to solve two step equations
- Understand that the solution to two linear equations is one of the following:
  - one solution – equations cross at one point (ordered pair)
  - infinitely many solutions – equations are equivalent
  - no solutions – equations do not cross
- Understand to solve for the x-variable of the solution of two linear equations set the two equations equal to each other and solve for the variable x
- Understand to solve for the y-variable of the solution of two linear equations plug the x-value back into either equation and solve for the variable y

MA.912.AR.9.2

Given a mathematical or real-world context, solve a system consisting of a two-variable linear equation and a

	<p>non-linear equation algebraically or graphically.</p> <p><b>Access Point</b>  MA.912.AR.9.AP.2 Solve a system consisting of a two-variable linear equation and a quadratic equation algebraically or graphically.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: slope, y-intercept, x-intercept, quadratic function, calculate graphically, calculate algebraically, linear equation, vertex, variable.</li> <li>• Understand how to graph a linear equation by using slope and y-intercept.</li> <li>• Understand how to graph a quadratic function by graphing the key features (Ex. vertex, y-intercept, x-intercept, etc.)</li> <li>• Understand solving a system consisting of linear and quadratic functions can be calculated either graphically or algebraically where the two functions cross.</li> <li>• Understand to solve a system consisting of linear and quadratic functions algebraically, set the two equations equal to each other and solve for the variables.</li> </ul>
MA.912.AR.9.3	<p>Given a mathematical or real-world context, solve a system consisting of two-variable linear or non-linear equations algebraically or graphically.</p> <p><b>Access Point</b>  MA.912.AR.9.AP.3 Solve a system consisting of two-variable linear or quadratic equations algebraically or graphically.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: slope, y-intercept, x-intercept, quadratic function, calculate graphically, calculate algebraically, linear equation, vertex, variable.</li> <li>• Understand how to graph a linear equation by using slope and y-intercept.</li> <li>• Understand how to graph a quadratic function by graphing the key features (Ex. vertex, y-intercept, x-intercept, etc.).</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand solving a system consisting of linear or quadratic functions can be calculated either graphically or algebraically where the two functions cross.</li> <li>• Understand to solve a system consisting of linear or quadratic functions algebraically, set the two equations equal to each other and solve for the variables.</li> </ul>
MA.912.AR.9.4	<p>Graph the solution set of a system of two-variable linear inequalities.</p> <p><b>Access Point</b> MA.912.AR.9.AP.4 Select the graph of the solution set of a system of two-variable linear inequalities.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following related vocabulary: boundary line, two-variable linear inequality, slope (m), y-intercept (b), graph, shading a graph, add (+), subtract (-), multiply (x), divide (<math>\div</math>), equal (=), Greater than (&gt;), Less than (&lt;), greater than or equal to (<math>\geq</math>), less than or equal to (<math>\leq</math>), variables, coordinate point, x-axis, y-axis, horizontal, vertical, solution</li> <li>• Understand the slope (rise over run) and the y-intercept (where the line crosses the y-axis, or <math>x =</math></li> </ul>
MA.912.AR.9.5	<p>Graph the solution set of a system of two-variable inequalities.</p> <p><b>Access Point</b> MA.912.AR.9.AP.5 Select the graph of the solution set of a system of two-variable inequalities.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: boundary line, two-variable inequality, graph, shading a graph, Greater than (&gt;), Less than (&lt;), greater than or equal to (<math>\geq</math>), less than or equal to (<math>\leq</math>), variables, coordinate point, x-axis, y-axis, horizontal, vertical, solution</li> <li>• Understand key features are used to graph</li> </ul>

an inequality function.

- Understand that a dotted boundary line on a graph of a two-variable inequality represents less than ( $<$ ) or greater than ( $>$ )
- Understand that a solid boundary line on a graph of a two-variable inequality represents less than or equal to ( $\leq$ ) or greater than or equal to ( $\geq$ )
- Understand if the graph of a two-variable inequality is shaded above the boundary line, the graph represents greater than or greater than or equal to
- Understand if the graph of a two-variable inequality is shaded below the boundary line, the graph represents less than or less than or equal to
- Understand that an inequality divides the coordinate plane into two parts by a boundary line where one represents the solutions of the inequality (Any coordinate point that falls in the shaded region or on the boundary line if it is solid line, is the solution.)
- Understand when given more than one two-variable inequalities, the solution is where the two shaded regions overlap.
- Understand when given more than one two-variable inequalities, if the two inequalities do not overlap, there is no solution.

### 9-12 Functions Strand

#### ***MA.912.F.1 Understand, compare and analyze properties of functions.***

MA.912.F.1.1

Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

MA.912.F.1.AP.1a Given an equation or graph that defines a function, identify the function type as either linear, quadratic, or exponential.

#### **Essential Understandings:**

- Understand the following terms and vocabulary: linear function, quadratic function, exponential function, graph, x-axis, y-axis, rapidly increase,

rapidly decrease, y-intercept, variable, slope, ratio, constant, parabola, line, curve

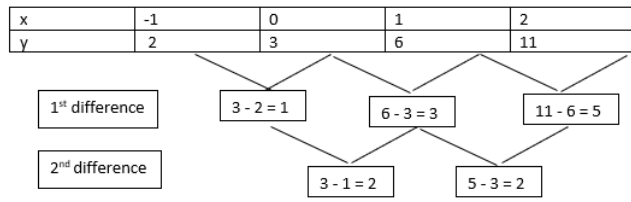
- Understand that a linear function is in the form of  $y = mx + b$  where  $m$  is the slope and  $b$  is the y-intercept
- Understand that a quadratic function is in the form of  $y = ax^2 + bx + c$  where the variable  $a \neq 0$ , and the variable  $c$  is the constant
- Understand that an exponential function is in the form  $y = ab^x$  where the variable  $a$  represents the initial value and the variable  $b$  represents the ratio between the y-values ( $a \neq 0, b \neq 1, \text{ and } b > 0$ )
- Understand that the graph of a quadratic function is a parabola
- Understand that the graph of a linear function is a line
- Understand that the graph of an exponential function is a curve that increases rapidly from left to right or decreases rapidly from left to right

MA.912.F.1.AP.1b Given an input-output table with an accompanying graph, determine a function type, either linear, quadratic, or exponential that could represent it.

**Essential Understandings:**

- Understand the following terms and vocabulary: graph, input-output table, linear function, x-values, y-values, common ratio, constant value, table, quadratic function, exponential function, 1<sup>st</sup> difference, 2<sup>nd</sup> difference, parabola, rapidly increase, rapidly decrease, line, curve
- Understand to determine that a given table is an exponential function, the x-values will increase by a constant value and the y-values will increase by a common ratio
- Understand to determine that a given table is a linear function, the x-values will increase by a constant value and the y-values will increase by a constant value
- Understand to determine that a given table

is a quadratic function, the 1<sup>st</sup> difference when subtracting the y-values will be different numbers, then when subtracting the new differences, the 2<sup>nd</sup> difference will be the same number



- Understand that the graph of a quadratic function is a parabola
- Understand that the graph of a linear function is a line
- Understand that the graph of an exponential function is a curve that increases rapidly from left to right or decreases rapidly from left to right

MA.912.F.1.2

Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

**Access Point**

MA.912.F.1.AP.2 Given a function represented in function notation, evaluate the function for an input in its domain.

**Essential Understandings:**

- Understand the following terms and vocabulary: function, function notation, input, domain, x-values, evaluate
- Understand that in function notation the  $f(x) = y$
- Understand that the inputs are the set of x-values
- Understand that the domain is the set of x-values
- Understand that evaluating a function means to plug the x-values into the function

MA.912.F.1.3

Calculate and interpret the average rate of change of a real-world situation represented graphically,



	<p>algebraically or in a table over a specified interval.</p> <p><b>Access Point</b></p> <p>MA.912.F.1.AP.3 Given a real-world situation represented graphically or algebraically, identify the rate of change as positive, negative, zero or undefined.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: rate of change, y-intercept (b), slope (m), slope intercept form, linear, negative slope, positive slope, coefficient, vertical, horizontal, x-axis, y-axis, variable, zero slope, undefined slope, increasing, decreasing.</li> <li>• Understand that slope is rise over the run.</li> <li>• Understand that slope is the rate of change.</li> <li>• Understand when identifying a rate of change, the rate of change is positive when the y-values increase as the x-values increase (the line is sloping upward from left to right).</li> <li>• Understand when identifying a rate of change, the rate of change is negative when the y-values decrease as the x-values increase (the line is sloping downward from left to right)</li> <li>• Understand when identifying a rate of change, the rate of change is zero when the y-values remain the same as the x-values increase (the line is horizontal)</li> <li>• Understand the rate of change is undefined when the y-values are different values, but the x-value remains the same (the line is vertical)</li> <li>• Understand that the slope intercept form is <math>y = mx + b</math>.</li> <li>• Understand the y-intercept is where the line crosses the y-axis (variable b).</li> <li>• Understand that in a linear equation the coefficient of the x-value is the slope (variable m).</li> </ul>
MA.912.F.1.5	<p>Compare key features of linear and nonlinear functions each represented in the same way, such as algebraically, graphically, in tables or written descriptions.</p> <p><b>Access Point</b></p>

MA.912.F.1.AP.5 Identify key features of linear and quadratic functions each represented in the same way algebraically or graphically (key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior).

**Essential Understandings:**

- Understand the following terms and vocabulary: linear function, quadratic function, graph, x-axis, y-axis, x-intercept, y-intercept, interval, increasing, decreasing, positive, negative, domain, range, positive infinity, negative infinity, upward, downward, slope, set.
- Understand a graph is read from left to right.
- Understand the y-intercept is where the function crosses the y-axis.
- Understand the x-intercept is where the function crosses the x-axis.
- Understand that an interval always refers to the x-values.
- Understand the function is increasing in the interval when the x-values increase, and the y-values increase.
- Understand the function is decreasing in the interval when the x-values increase, and the y-values decrease.
- Understand that the domain is the set of all the x-values.
- Understand that the range is the set of all the y-values.
- Understand in a quadratic function that is opening upward, as the x-values decrease the y-values increase to positive infinity.
- Understand in a quadratic function that is opening upward, as the x-values increase, the y-values increase to positive infinity.
- Understand in a quadratic function that is opening downward, as the x-values decrease, the y-values decrease to negative infinity.
- Understand in a quadratic function that is opening downward, as the x-values increase

	<p>the <math>y</math>-values decrease to negative infinity.</p> <ul style="list-style-type: none"> <li>• Understand in a linear function, if the slope is positive the function will go upward from left to right.</li> <li>• Understand in a linear function, if the slope is negative the function will go downward from left to right.</li> </ul>
<p>MA.912.F.1.6</p>	<p>Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.</p> <p><b>Access Point</b></p> <p>MA.912.F.1.AP.6 Identify key features of linear, quadratic or exponential functions each represented in a different way algebraically or graphically (key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior).</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: linear function, quadratic function, exponential function, graph, <math>x</math>-axis, <math>y</math>-axis, <math>x</math>-intercept, <math>y</math>-intercept, interval, increasing, decreasing, positive, negative, domain, range, positive infinity, negative infinity, upward, downward, slope, set, growth, decay, exponentially.</li> <li>• Understand a graph is read from left to right.</li> <li>• Understand the <math>y</math>-intercept is where the function crosses the <math>y</math>-axis.</li> <li>• Understand the <math>x</math>-intercept is where the function crosses the <math>x</math>-axis.</li> <li>• Understand that an interval always refers to the <math>x</math>-values.</li> <li>• Understand the function is increasing in the interval when the <math>x</math>-values increase, and the <math>y</math>-values increase.</li> <li>• Understand the function is decreasing in the interval when the <math>x</math>-values increase, and the <math>y</math>-values decrease.</li> <li>• Understand that the domain is the set of all the <math>x</math>-values.</li> <li>• Understand that the range is the set of all the</li> </ul>

	<p>y-values.</p> <ul style="list-style-type: none"> <li>• Understand in a quadratic function that is opening upward, as the x-values decrease the y-values increase to positive infinity.</li> <li>• Understand in a quadratic function that is opening upward, as the x-values increase, the y-values increase to positive infinity.</li> <li>• Understand in a quadratic function that is opening downward, as the x-values decrease, the y-values decrease to negative infinity.</li> <li>• Understand in a quadratic function that is opening downward, as the x-values increase the y-values decrease to negative infinity.</li> <li>• Understand in a linear function, if the slope is positive the function will go upward from left to right.</li> <li>• Understand in a linear function, if the slope is negative the function will go downward from left to right.</li> <li>• Understand for exponential growth, as x increases, y grows exponentially (to positive infinity).</li> <li>• Understand for exponential decay, as x increases, y decreases exponentially (to negative infinity).</li> </ul>
<p>MA.912.F.1.7</p>	<p>Compare key features of two functions each represented algebraically, graphically, in tables or written descriptions.</p> <p><b>Access Point</b></p> <p>MA.912.F.1.AP.7 Compare key features of two functions each represented algebraically or graphically.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: linear function, quadratic function, graph, x-axis, y-axis, x-intercept, y-intercept, interval, increasing, decreasing, positive, negative, domain, range, positive infinity, negative infinity, slope, end behavior, vertex, maximum, minimum.</li> <li>• Understand a graph is read from left to right.</li> <li>• Understand key features can include some of</li> </ul>

the following depending on the function:  $x$ -intercept,  $y$ -intercept, slope, increasing/decreasing intervals, vertex, relative maximum/minimum values, end behavior, domain, range, etc.

- Understand the  $y$ -intercept is where the function crosses the  $y$ -axis.
- Understand the  $x$ -intercept is where the function crosses the  $x$ -axis.
- Understand that an interval always refers to the  $x$ -values.
- Understand the function is increasing in the interval when the  $x$ -values increase, and the  $y$ -values increase.
- Understand the function is decreasing in the interval when the  $x$ -values increase, and the  $y$ -values decrease.
- Understand that the domain is the set of all the  $x$ -values.
- Understand that the range is the set of all the  $y$ -values.
- Understand that slope is how steep a linear function is.
- Understand the vertex of a quadratic is the maximum or minimum point.
- Understand that the relative maximums are where the function changes from increasing to decreasing.
- Understand that the relative minimums are where the function changes from decreasing to increasing.
- Understand the end behavior refers to what the graph is doing as it approaches negative infinity or positive infinity.

MA.912.F.1.8

Determine whether a linear, quadratic or exponential function best models a given real-world situation.

**Access Point**

MA.912.F.1.AP.8 Select whether a linear or quadratic function best models a given real-world situation.

**Essential Understandings:**

	<ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: linear function, quadratic function, rate of change, parabola, line.</li> <li>• Understand that a linear function models behavior that forms a line (ex. any problem that involves a rate of change)</li> <li>• Understand that a quadratic function models behavior that forms a parabola (ex: throwing a ball upward, water coming out of a fountain, etc.)</li> </ul>
MA.912.F.1.9	<p>Determine whether a function is even, odd or neither when represented algebraically, graphically or in a table.</p> <p><b>Access Point</b></p> <p>MA.912.F.1.AP.9 Select whether a function is even, odd or neither when represented algebraically.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: function, even, odd, algebraically, negative, positive, opposite sign.</li> <li>• Understand when <math>f(-x)=f(x)</math> the function is even. <ul style="list-style-type: none"> <li>Ex. <math>f(x)=x^2+5</math> is even because plugging in a negative <math>x</math> will not change the function</li> </ul> </li> <li>• Understand when <math>f(-x)=-f(x)</math> the function is odd. <ul style="list-style-type: none"> <li>Ex. <math>f(x)=x^3-x</math> is odd because plugging in a negative <math>x</math> will change all the signs to the opposite sign</li> </ul> </li> <li>• Understand the function is neither if it does not fit the rule for even or odd. <ul style="list-style-type: none"> <li>Ex. <math>f(x)=x^2+x-3</math> is neither because plugging in a negative <math>x</math> will change some of the signs but not all</li> </ul> </li> </ul>
<p><b>MA.912.F.2 Identify and describe the effects of transformations on functions. Create new functions given transformation.</b></p>	
MA.912.F.2.1	<p>Identify the effect on the graph or table of a given function after replacing <math>(x)</math> by <math>(x) + k</math>, <math>(x-k)</math>, <math>(kx)</math> and <math>f(x+k)</math> for specific values of <math>k</math>.</p> <p><b>Access Point</b></p>

	<p>MA.912.F.2.AP.1 Select the effect (up, down, left, or right) on the graph of a given function after replacing <math>f(x)</math> by <math>f(x) + k</math> and <math>f(x + k)</math> for specific values of <math>k</math>.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: transforming, shifting, graph, x-axis, y-axis, left, right, upward, downward, positive, negative, function, addition (+), subtraction (-), integer.</li> <li>• Understand how to add and subtract integers.</li> <li>• Understand when transforming <math>f(x) + k</math>, adding a positive value for <math>k</math>, shifts the entire graph upward (ex. <math>x^2 + 3</math>, the function <math>x^2</math> is shifted up 3 places)</li> <li>• Understand when transforming <math>f(x) + k</math>, adding a negative value (or subtracting a value) for <math>k</math>, shifts the entire graph downward (ex. <math>x^2 - 3</math>, the function <math>x^2</math> is shifted down 3 places)</li> <li>• Understand when transforming <math>f(x + k)</math>, adding a positive value for <math>k</math>, shifts the entire graph to the left (ex. <math>(x + 3)^2</math>, the function <math>x^2</math> is shifted over 3 places to the left)</li> <li>• Understand when transforming <math>f(x + k)</math>, adding a negative value (or subtracting a value) for <math>k</math>, shifts the entire graph to the right (ex. <math>(x - 3)^2</math>, the function <math>x^2</math> is shifted over 3 places to the right)</li> </ul>
<p>MA.912.F.2.2</p>	<p>Identify the effect on the graph of a given function of two or more transformations defined by adding a real number to the x- or y- values or multiplying the x- or y-values by a real number.</p> <p><b>Access Point</b></p> <p>Identify the effect on the graph of a given function of two or more transformations defined by adding a real number to the x- or y-values.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: function, transformation, real number, positive, negative, up, down, left, right, x-value, y-value.</li> <li>• Understand that adding a positive real number to the x-values will move the graph left.</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that adding a negative real number to the <math>x</math>-values will move the graph to the right.</li> <li>• Understand that adding a positive real number to the <math>y</math>-values will move the graph up.</li> <li>• Understand that adding a negative real number to the <math>y</math>-values will move the graph down</li> </ul>
MA.912.F.2.3	<p>Given the graph or table of <math>f(x)</math> and the graph or table of <math>f(x)+k, kf(x), f(kx)</math> and <math>f(x+k)</math>, state the type of transformation and find the value of the real number <math>k</math>.</p> <p><b>Access Point</b></p> <p>MA.912.F.2.AP.3 Given the graph of a given function after replacing <math>f(x)</math> by <math>f(x) + k</math> and <math>f(x + k), kf(x)</math>, for specific values of <math>k</math> select the type of transformation and find the value of the real number <math>k</math>.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: function, transformation, real number, positive, negative, up, down, left, right, <math>x</math>-value, <math>y</math>-value, <math>k</math>-value, horizontally, vertically, shrink, stretch, adding, multiplying.</li> <li>• Understand that adding a positive <math>k</math>-value to the <math>x</math>-values will move the graph left.</li> <li>• Understand that adding a negative <math>k</math>-value to the <math>x</math>-values will move the graph to the right.</li> <li>• Understand that adding a positive <math>k</math>-value to the <math>y</math>-values will move the graph up.</li> <li>• Understand that adding a negative <math>k</math>-value to the <math>y</math>-values will move the graph down.</li> <li>• Understand that multiplying by a <math>k</math>-value where <math>k &gt; 1</math> will stretch the graph vertically.</li> <li>• Understand that multiplying by a <math>k</math>-value where <math>0 &lt; k &lt; 1</math> will shrink the graph vertically.</li> </ul>
MA.912.F.2.5	<p>Given a table, equation or graph that represents a function, create a corresponding table, equation or graph of the transformed function defined by adding a real number to the <math>x</math>- or <math>y</math>-values or multiplying the <math>x</math>- or <math>y</math>-values by a real number.</p> <p><b>Access Point</b></p>



MA.912.F.2.AP.5 Given a table, equation or graph that represents a function, select a corresponding table, equation or graph of the transformed function defined by adding a real number to the x- or y-values.

**Essential Understandings:**

- Understand the following terms and vocabulary: function, transformation, table, equation, graph, real number, positive, negative, up, down, left, right, x-value, y-value, horizontal, vertical.
- Understand that adding a positive real number to the x-values will move the graph left.
- Understand that adding a negative real number to the x-values will move the graph to the right.
- Understand that adding a positive real number to the y-values will move the graph up.
- Understand that adding a negative real number to the y-values will move the graph down .
- Understand that  $y=f(x+k)$  is the rule for adding a real number to the x-values
- Understand when examining a table, to determine horizontal movement right and left, look at the x-values (Ex.  $y=f(x+4)$  shifts to the left 4.)
- Understand that  $y=f(x)+k$  is the rule for adding a real number to the y-values
- Understand when examining a table, to determine vertical movement up and down, look at the y-values.
- If the y-values decrease, k will be negative.
- If the y-values increase, k will be positive.

**MA.912.F.3 Create new functions from existing functions.**

MA.912.F.3.2

Given a mathematical or real-world context, combine two or more functions, limited to linear, quadratic, exponential and polynomial, using arithmetic operations. When appropriate, include domain restrictions for the

	<p>new function.</p> <p><b>Access Point</b></p> <p>MA.912.F.3.AP.2 Given a mathematical and/or real-world context, combine two or more functions, limited to linear, quadratic, and polynomial, using arithmetic operations of addition, subtraction, or multiplication.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: function, linear, quadratic, polynomial, like terms, distributive property, product, sum, difference.</li> <li>• Understand the following rules:</li> <li>• Sum: <math>(f+g)(x)=f(x)+g(x)</math>.</li> <li>• Difference: <math>(f-g)(x)=f(x)-g(x)</math>.</li> <li>• Product: <math>(f*g)(x)=f(x)g(x)</math>.</li> <li>• Understand that when adding two or more functions, add or subtract like terms.</li> <li>• Understand when subtracting two or more functions, the distributive property will need to be applied to any function that is subtracted then add or subtract like terms.</li> <li>• Understand when multiplying two or more functions, the distributive property will need to be applied then add or subtract like terms.</li> </ul>
MA.912.F.3.4	<p>Represent the composition of two functions algebraically or in a table. Determine the domain and range of the composite function.</p> <p><b>Access Point</b></p> <p>MA.912.F.3.AP.4 Given a composite function within a mathematical or real-world context, identify the domain and range of the composite function.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: composite function, inside function, domain, range, x-values, y-values, substitute.</li> <li>• Understand the following rule:</li> <li>• <math>(f\circ g)(x)=f(g(x))</math></li> </ul>

- Understand for composite functions, using the rule above, the function of  $g$  is substituted into every  $x$  in the function of  $f$ .
- Understand the following rule:  
 $(g \circ f)(x) = g(f(x))$
- Understand for composite functions, using the rule above, the function of  $f$  is substituted into every  $x$  in the function of  $g$ .
- Understand the domain is all the  $x$ -values and the range is all the  $y$ -values.
- Understand that the domain of a composite function is where the domain of the composite function and the inside function overlap.
- Understand for the rule  $(f \circ g)(x) = f(g(x))$ ,  $g(x)$  is the inside function.
- Understand for the rule  $(g \circ f)(x) = g(f(x))$ ,  $f(x)$  is the inside function.

MA.912.F.3.6

Determine whether an inverse function exists by analyzing tables, graphs and equations.

**Access Point**

MA.912.F.3.AP.6 Determine whether an inverse function exists by analyzing graphs and equations.

**Essential Understandings:**

- Understand the following terms and vocabulary: inverse function, one-to-one relationship, reflect, horizontal line test, domain, graph.
- Understand when analyzing a graph, a function and its inverse will reflect over the line  $y=x$
- Understand that a function and its inverse have a one-to-one relationship (Ex. because they reflect over the line  $y=x$ , if  $x$  is 3,  $y$  is 3 etc.)
- Understand that a function and its inverse follow these rules:

	$f(g(x))=x$ $g(f(x))=x$ <ul style="list-style-type: none"> <li>• Understand that a function must pass the horizontal line test in order to have an inverse.</li> <li>• Understand that some functions will have an inverse if the domain is isolated (Ex. <math>y=x^2</math> does not pass the horizontal line test unless the domain is isolated, <math>x \geq 0</math> or <math>x \leq 0</math>)</li> </ul>
MA.912.F.3.7	<p>Represent the inverse of a function algebraically, graphically or in a table. Use composition of functions to verify that one function is the inverse of the other.</p> <p><b>Access Point</b></p> <p>MA.912.F.3.AP.7 Represent the inverse of a function algebraically. Use composition of functions to verify that one function is the inverse of the other.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: inverse function, one-to-one relationship, reflect, horizontal line test, domain, graph.</li> <li>• Understand when analyzing a graph, a function and its inverse will reflect over the line <math>y = x</math></li> <li>• Understand that a function and its inverse have a one-to-one relationship (Ex. because they reflect over the line <math>y = x</math>, if <math>x</math> is 3, <math>y</math> is 3 etc.)</li> <li>• Understand that a function and its inverse follow these rules: <math display="block">f(g(x)) = x</math> <math display="block">g(f(x)) = x</math> </li> </ul> <ul style="list-style-type: none"> <li>• Understand that a function must pass the horizontal line test in order to have an inverse.</li> <li>• Understand that some functions will have an inverse if the domain is isolated (Ex. <math>y = x^2</math> does not pass the horizontal line test unless the domain is isolated, <math>x \geq 0</math> or <math>x \leq 0</math>)</li> </ul>

**9-12 Financial Literacy Strand**

<b>MA.912.FL.1 Build mathematical foundations for financial literacy.</b>	
MA.912.FL.1.1.	Extend previous knowledge of operations of fractions, percentages and decimals to solve real-world problems involving money and business.
	<b>Access Point</b> MA.912.FL.1.AP.1 Solve real-world problems involving money using percentages and decimals.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: discounts, sales tax, tips, addition, subtraction, multiplication, calculator, price, percentage, decimal, rational numbers.</li> <li>• Understand how to convert from percentage to decimal and decimal to percentage.</li> <li>• Understand how to add, subtract and multiply rational numbers.</li> <li>• Understand how to calculate sales tax.</li> <li>• Understand how to estimate a tip.</li> <li>• Understand how to calculate a discounted price.</li> </ul>
MA.912.FL.1.2	Extend previous knowledge of ratios and proportional relationships to solve real-world problems involving money and business.
	<b>Access Point</b> MA.912.FL.1.AP.2 Solve simple real-world problems involving money using ratios or proportions.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: ratio, proportion, addition, subtraction multiplication, division, rational numbers, simplify, cost, quantity.</li> <li>• Understand how to multiply and divide rational numbers.</li> <li>• Understand how to solve a proportion.</li> <li>• Understand how to simplify a ratio.</li> <li>• Understand how to set up a ratio ( <math>\frac{cost}{quantity}</math> )</li> </ul>
<b>MA.912.FL.2 Develop an understanding of basic accounting and economic principles.</b>	
MA.912.FL.2.2	Solve real-world problems involving profits, costs and revenues using spreadsheets and other technology.
	<b>Access Point</b> MA.912.FL.2.AP.2 Calculate the profit when given the expenses and revenue from a real-world problem.

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: profit, cost, revenue, subtraction, rational numbers.</li> <li>• Understand which value represents the cost, the profit and the revenue in a real-world problem or spreadsheet.</li> <li>• Understand how to subtract rational numbers.</li> <li>• Understand that cost is subtracted from the revenue to determine profit.</li> </ul>
MA.912.FL.2.3	<p>Explain how consumer price index (CPI), gross domestic product (GDP), stock indices, unemployment rate and trade deficit are calculated. Interpret their value in terms of the context.</p> <p><b>Access Point</b> MA.912.FL.2.AP.3 Given the consumer price index (CPI), stock indices, or unemployment rates for two different time periods, identify whether the rates are increasing or decreasing.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Compare unemployment rate from two years.</li> <li>• Cost of a loaf of bread in 1963 was \$.22, and today it cost \$3.00 (first concept of inflation).</li> </ul>
MA.912.FL.2.4	<p>Given current exchange rates, convert between currencies. Solve real-world problems involving exchange rates.</p> <p><b>Access Point</b> MA.912.FL.2.AP.4 Given current exchange rates, convert between currencies.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: exchange rate, currency, multiplication, division, rational numbers, convert.</li> <li>• Understand how to multiply and divide rational numbers.</li> <li>• Understand <math>c = (a)(b)</math>  c = money after  a = money you have  b = exchange rate</li> </ul>
MA.912.FL.2.5	<p>Develop budgets that fit within various incomes using spreadsheets and other technology.</p> <p><b>Access Point</b></p>

	<p>MA.912.FL.2.AP.5 Given typical monthly expenses (housing, utilities, food, etc.), determine the monthly income needed.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: expenses, income, addition, rational numbers, greater than, less than, monthly.</li> <li>• Understand how to add rational numbers.</li> <li>• Understand to determine the monthly income needed, all expenses must be added together.</li> <li>• Understand that the monthly income must be greater than the monthly expenses.</li> </ul>
<p>MA.912.FL.2.6</p>	<p>Given a real-world scenario, complete and calculate federal income tax using spreadsheets and other technology.</p> <p><b>Access Point</b> MA.912.FL.2.AP.6 Given a paycheck, identify the taxes taken out.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: taxes, gross earned income, net income.</li> <li>• Understand that taxes are taken out of earned income.</li> <li>• Understand the difference between gross and net income.</li> </ul>
<p><b>MA.912.FL.3 Describe the advantages and disadvantages of short-term and long-term purchases.</b></p>	
<p>MA.912.FL.3.1</p>	<p>Compare simple, compound and continuously compounded interest over time.</p> <p><b>Access Point</b> MA.912.FL.3.AP.1 Compare simple and compound interest over time.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: rate, interest, principal, time, number of times compounded, percentages, decimals, real numbers, multiplication (x), compound interest, simple interest, variables, formulas, final amount.</li> <li>• Understand the difference between simple and compound interest.</li> <li>• Understand that rate (<math>r</math>) is always in decimal form. (Ex. 6% will be expressed in the formula as 0.06.)</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that simple interest is interest paid on the principal only over a period of time. (Ex. Car loans, most bank loans).</li> <li>• Understand to calculate simple interest use the formula <math>I = Prt</math> (where I = interest, P = principal, r = rate, t = time).</li> <li>• Understand that in a simple or a compound interest problem, time(t) is in terms of years. (Ex. 3 months: <math>\frac{3}{12} = t</math>).</li> <li>• Understand that compound interest is interest paid on the initial principal plus interest on the interest charged previously. (Ex. Credit cards, savings account)</li> <li>• Understand to calculate compound interest use the formula <math>A = P(1 + \frac{r}{n})^{nt}</math> (where A = final amount, P = principal, r = rate, t = time, n = number of times compounded).</li> </ul>
MA.912.FL.3.2	<p>Solve real-world problems involving simple, compound and continuously compounded interest.</p> <p><b>Access Point</b> MA.912.FL.3.AP.2 Solve real-world problems involving simple and compound interest.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: rate, interest, principal, time, number of times compounded, percentages, decimals, real numbers, multiplication (x), compound interest, simple interest, variables, formulas, final amount.</li> <li>• Understand how to convert percentages to decimals.</li> <li>• Understand how to multiply real numbers with a calculator.</li> <li>• Understand in equations when variables are side by side with no sign between them it is implied that the values are multiplied (ex. <math>Prt</math> means <math>(P)(r)(t)</math> or P times r times t)</li> <li>• Understand that rate (<math>r</math>) is always in decimal form (ex. 6% will be expressed in the formula as 0.06.)</li> <li>• Understand that simple interest is interest paid</li> </ul>



	<p>on the principal only over a period of time (ex. Car loans, most bank loans)</p> <ul style="list-style-type: none"> <li>• Understand to calculate simple interest use the formula <math>I = Prt</math> (where I = interest, P = principal, r = rate, t = time)</li> <li>• Understand that in a simple or a compound interest problem, time(t) is in terms of years (ex. 3 months: <math>\frac{3}{12} = t</math>)</li> <li>• Understand that compound interest is interest paid on the initial principal plus interest on the interest charged previously (ex. Credit cards, savings account)</li> <li>• Understand to calculate compound interest use the formula <math>A = P(1 + \frac{r}{n})^{nt}</math> (where A = final amount, P = principal, r = rate, t = time, n = number of times compounded)</li> </ul>
MA.912.FL.3.4	<p>Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.</p> <p><b>Access Point</b></p> <p>MA.912.FL.3.AP.4 Identify the relationship between simple interest and linear growth. Identify the relationship between compound interest and exponential growth.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: linear growth, exponential growth, simple interest, compound interest, percentage, constant proportion, value.</li> <li>• Understand that linear growth is a slow and steady growth and exponential growth is a rapid and steep growth.</li> <li>• Understand that simple interest problems show linear growth.</li> <li>• Understand that linear growth is growing by the same amount over a period of time.</li> <li>• Understand that simple interest problems grow by the same percentage each year (linear growth)</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that exponential growth is growth that increases quickly over time.</li> <li>• Understand that compound interest problems show exponential growth.</li> <li>• Understand that exponential growth is growing in increasing value (constant proportion) over time.</li> <li>• Understand that compound interest problems grow by a constant proportion over time (exponential growth).</li> </ul>
MA.912.FL.3.5	Compare the advantages and disadvantages of using cash versus personal financing options.
	<p><b>Access Point</b> MA.912.FL.3.AP.5 Select the advantages and disadvantages of using cash versus credit.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: cash, credit card, security breaches, personal information, overspending, convenient, credit, interest payment, fraud, reward points, emergency funds, budget, fees.</li> <li>• Understand the advantages to using cash can be there are no security breaches, protects your personal information, more difficult to give away than using credit card, no overspending, convenient, etc.</li> <li>• Understand that the advantages of using a credit card can be that it builds credit, can pay overtime, fraud protection, earn reward points, emergency funds, convenient, way to budget purchases, etc.</li> <li>• Understand the disadvantages to using cash can be it can be easily lost or stolen, some places don't except cash, ATM withdrawal fees to get cash, not building credit, can't make online purchases, etc.</li> <li>• Understand the disadvantages to using a credit card can be interest and fees can be charged if balance is not paid in full each month, easier to overspend on purchases or make unnecessary purchases, can</li> </ul>

	negatively affect credit scores, etc.
MA.912.FL.3.6	<p>Calculate the finance charges and total amount due on a bill using various forms of credit using estimation, spreadsheets and other technology.</p> <p><b>Access Point</b> MA.912.FL.3.AP.6 Given a bill statement, identify the finance charge, interest rate and total amount due.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: bill statement, finance charge, interest rate, amount due,</li> <li>• Understand the difference between a finance charge and an interest rate.</li> </ul>
MA.912.FL.3.7	<p>Compare the advantages and disadvantages of different types of student loans by manipulating a variety of variables and calculating the total cost using spreadsheets and other technology.</p> <p><b>Access Point</b> MA.912.FL.3.AP.7 Given two different student loan options, compare the advantages and disadvantages of each loan's interest rate, monthly payment and total cost.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: interest rate, monthly payment, student loan, total cost, principal, borrower, lender, advantage, disadvantage, percentage, length of loan.</li> <li>• Understand that a student loan is money borrowed to fund higher education.</li> <li>• Understand that the interest is the amount of additional money the lender receives for loaning the borrower money.</li> <li>• Understand that the interest rate is a percentage of the loan amount (principal)</li> <li>• Understand that the interest rate, length of loan, and monthly payment determine the total cost of a student loan.</li> <li>• Understand that there are advantages and disadvantages to borrowing money (different interest rates, higher and lower monthly payments,</li> </ul>

	length of loan, and total cost).
MA.912.FL.3.8	<p>Calculate using spreadsheets and other technology the total cost of purchasing consumer durables over time given different monthly payments, down payments, financing options and fees.</p> <p><b>Access Point</b> MA.912.FL.3.AP.8 Given the total cost of an item purchased using two different payment plans, calculate the total cost difference of the item between payment plans.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: payment plan, total cost, interest rate, loan amount, principal, length of loan, down payment, monthly payment, subtraction.</li> <li>• Understand that the payment plan includes interest rate, loan amount (principal), length of loan, possible down payment, and monthly payment.</li> <li>• Understand that the interest rate, length of loan, possible down payment and monthly payment determine the total cost the item purchased.</li> <li>• Understand to calculate the total cost difference, subtract the lower amount from the higher amount.</li> </ul>
MA.912.FL.3.9	<p>Compare the advantages and disadvantages of different types of mortgage loans by manipulating a variety of variables and calculating fees and total cost using spreadsheets and other technology.</p> <p><b>Access Point</b> MA.912.FL.3.AP.9 Given two different mortgage loans, one 15-year and one 30-year, compare the advantages and disadvantages of each loan’s interest rate, monthly payment and total cost.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: mortgage loan, interest rate, monthly payment, total cost, accrue.</li> <li>• Understand the 30-year mortgage loan accrues more interest which will increase the overall total cost.</li> <li>• Understand the 15-year mortgage loan accrues less interest which will lower the overall total cost.</li> <li>• Understand that the 30-year mortgage loan will</li> </ul>

	<p>have a lower monthly payment.</p> <ul style="list-style-type: none"> <li>• Understand that the 15-year mortgage loan will have a higher monthly payment.</li> <li>• Understand the larger the interest rate, the higher the monthly payment, and higher total cost.</li> </ul>
MA.912.FL.3.10	<p>Analyze credit scores qualitatively. Explain how short-term and long-term purchases, including deferred payments, may increase or decrease credit scores. Explain how credit scores influence buying power.</p> <p><b>Access Point</b> MA.912.FL.3.AP.10 Identify how short-term and long-term purchases, past payment history, and amount of debt may increase or decrease credit scores.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: short-term purchases, long-term purchases, debt, credit score, on-time payment, credit history, payment history.</li> <li>• Understand that a higher credit score is better than a lower credit score.</li> <li>• Understand the more purchases made with credit the lower the credit score.</li> <li>• Understand that a long-term purchase decreases credit scores.</li> <li>• Understand that on time payments increase credit scores.</li> <li>• Understand that the larger amount of debt the lower the credit score.</li> <li>• Understand that using short-term purchases can create credit history.</li> <li>• Understand where to find credit scores.</li> </ul>
MA.912.FL.3.11	<p>Given a real-world scenario, establish a plan to pay off debt.</p> <p><b>Access Point</b> MA.912.FL.3.AP.11 Given several payment plans, with the monthly payment calculated, select the plan that will reduce the debt the quickest.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: principal, monthly payment, payment plan, debt, interest rate, time.</li> <li>• Understand that the interest rate can affect the</li> </ul>

	<p>monthly payment.</p> <ul style="list-style-type: none"> <li>• Understand that the total number of payments can affect the monthly payment.</li> <li>• Understand the benefits of paying off a debt quicker.</li> <li>• Understand the parts of a payment plan. (Interest rate, time, principal, payment).</li> </ul>
<p><b>MA.912.FL.4 Describe the advantages and disadvantages of financial and investment plans, including insurances.</b></p>	
MA.912.FL.4.1	<p>Calculate and compare various options, deductibles and fees for various types of insurance policies using spreadsheets and other technology.</p> <p><b>Access Point</b>  MA.912.FL.4.AP.1 Compare various options, deductibles and fees for various types of individual insurance policies, such as medical, car and/or homeowners' insurance.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: deductible, fee, insurance policy, claim, coverage, more, less, co-pay, premiums, health history, driving record, car make and model.</li> <li>• Understand insurance policies offer different types of coverage.</li> <li>• Understand which insurance policy costs more/less.</li> <li>• Understand some insurance policies can cost more and offer less coverage.</li> <li>• Understand that it is best to get the most coverage for the least amount of money.</li> <li>• Understand that the deductible is paid each time a claim is made.</li> <li>• Understand there are factors that can affect coverage. (Age, health history, driving record, car make and model, etc.).</li> </ul>
MA.912.FL.4.3	<p>Compare the advantages and disadvantages of various retirement savings plans using spreadsheets and other technology.</p> <p><b>Access Point</b>  MA.912.FL.4.AP.3 List the advantages and disadvantages of having a retirement savings plan.</p> <p><b>Essential Understandings:</b></p>

	<ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: probability, events decimal form, conditional probability.</li> <li>• Understand that conditional probability is the probability of an event occurring given that we have prior information. (The probability of surviving the sinking of the titanic given that the person is a female. Females were put in the lifeboats first therefore more likely to survive the accident.)</li> <li>• Understand that the conditional probability of B given A is calculated by dividing the probability of A and B by the probability of A. <math>(P(B   A) = (P(A \text{ and } B)) / (P(A)))</math>.</li> </ul>
MA.912.FL.4.4	<p>Collect, organize and interpret data to determine an effective retirement savings plan to meet personal financial goals using spreadsheets and other technology.</p> <p><b>Access Point</b> MA.912.FL.4.AP.4 Select a retirement savings plan to meet a given personal financial goal.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: retirement, savings plan, financial goal, budget, IRA, pension plan, annuities,</li> <li>• Understand that a retirement plan is a savings plan for retirement years.</li> <li>• Understand how to budget for a financial retirement goal.</li> <li>• Understand there are several different retirement plan options. (IRA, pension, annuities.)</li> </ul>
MA.912.FL.4.5	<p>Compare different ways that portfolios can be diversified in investments.</p> <p><b>Access Point</b> MA.912.FL.4.AP.5 List an advantage of diversifying investments.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: diversify, investment, risk, asset, economic worth, invest.</li> <li>• Understand that an asset (cash, certificates of deposit, savings, money market accounts, etc.) is something of economic worth owned by an individual or group.</li> </ul>

	<ul style="list-style-type: none"> <li>Understand that assets can be invested in different ways to reduce risk.</li> </ul>
MA.912.FL.4.6	<p>Simulate the purchase of a stock portfolio with a set amount of money, and evaluate its worth over time considering gains, losses and selling, taking into account any associated fees.</p> <p><b>Access Point</b> MA.912.FL.4.AP.6 Simulate the buying and selling of a single stock and identify its worth over time.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand the following terms and vocabulary: independence, events, probability.</li> <li>Understand that independent events are events where the probability of one event does not affect the probability of another event. (Flipping a coin and rolling a die are independent because flipping the coin does not affect the probability of rolling a die).</li> </ul>

### 9-12 Geometric Reasoning Strand

<b><i>MA.912.GR.1 Prove and apply geometric theorems to solve problems.</i></b>	
MA.912.GR.1.1	<p>Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles.</p> <p><b>Access Point</b> MA.912.GR.1.AP.1 Use the relationships and theorems about lines and angles to solve mathematical or real-world problems involving postulates, relationships and theorems of lines and angles.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand the following terms and vocabulary: acute angle, obtuse angle, right angle, straight angle, vertical angles, parallel lines, perpendicular lines, adjacent angles, alternate interior angles, congruent angles, transversal, vertex, ray, protractor, supplementary angles, complimentary angles, horizontal lines, vertical lines.</li> <li>Understand that a protractor can be used to show that angles are supplementary (equals 180 degrees)</li> </ul>



	<ul style="list-style-type: none"> <li>• Understand that a protractor can be used to show that angles are equal.</li> <li>• Understand that the angle measure of a straight line is 180 degrees.</li> <li>• Understand that vertical angles, adjacent angles, and supplementary angles are formed when given a set of parallel lines cut by a third line called a transversal.</li> <li>• Understand that two angles can be supplementary (two angles that added together equal 180 degrees) or complimentary (two angles that added together equal 90 degrees).</li> <li>• Understand that pairs of congruent angles are formed when given a set of parallel lines cut by a transversal.</li> <li>• Understand that alternate interior angles are formed when given a set of parallel lines cut by a transversal.</li> <li>• Understand that there are several types of angles, included but not limited to: acute angle, right angle, obtuse angle, straight angle.</li> <li>• Understand that there are several types of lines, included but not limited to: Parallel lines, perpendicular lines, intersecting lines, vertical line, horizontal line, transversal line.</li> <li>• Understand that a line must contain at least two points.</li> <li>• Understand that an angle is formed when two rays intersect at a point called the vertex.</li> </ul>
MA.912.GR.1.2	<p>Prove triangle congruence or similarity using Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side, Angle-Angle and Hypotenuse-Leg.</p> <p><b>Access Point</b></p> <p>MA.912.GR.1.AP.2 Identify the triangle congruence or similarity criteria; Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side, Angle-Angle and Hypotenuse-Leg.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: congruent triangles, similar triangles, congruent angles, proportion, side-side-side (SSS), side-angle-side (SAS), angle-side-angle (ASA), angle-</li> </ul>

	<p>angle-side (AAS) and hypotenuse-leg (HL), angle-angle (AA).</p> <ul style="list-style-type: none"> <li>• Understand that two congruent triangles are two triangles that are the same shape and same size.</li> <li>• Understand that two similar triangles are two triangles whose sides are in proportion to each other, and their angles are congruent.</li> <li>• Understand that the following can be used to prove that two triangles are congruent: side-side-side (SSS), side-angle-side (SAS), angle-side-angle (ASA), angle-angle-side (AAS) and hypotenuse-leg (HL).</li> <li>• Understand that the following can be used to prove that two triangles are similar: angle-angle (AA), side-angle-side (SAS), and side-side-side (SSS).</li> </ul>
<p>MA.912.GR.1.3</p>	<p>Prove relationships and theorems about triangles. Solve mathematical and real-world problems involving postulates, relationships and theorems of triangles.</p> <p><b>Access Point</b></p> <p>MA.912.GR.1.AP.3 Use the relationships and theorems about triangles. Solve mathematical and/or real-world problems involving postulates, relationships and theorems of triangles.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: triangle, polygon, sides, angles, interior angles, equilateral triangle, isosceles triangle, scalene triangle, right triangle, acute triangle, obtuse triangle, congruent triangle, similar triangle, congruent angle, proportion.</li> <li>• Understand that a triangle is a polygon which consists of three sides.</li> <li>• Understand that a triangle consists of three angles.</li> <li>• Understand that when adding the three interior angles of a triangle the sum of the angles is 180 degrees.</li> <li>• Understand that triangles can be named by</li> </ul>

	<p>their sides: equilateral, isosceles, scalene.</p> <ul style="list-style-type: none"> <li>• Understand that triangles can be named by their angles: right triangle, acute triangle, obtuse triangle.</li> <li>• Understand that two congruent triangles are two triangles that are the same shape and same size.</li> <li>• Understand that two similar triangles are two triangles whose sides are in proportion to each other, and their angles are congruent.</li> </ul>
<p>MA.912.GR.1.4</p>	<p>Prove relationships and theorems about parallelograms. Solve mathematical and real-world problems involving postulates, relationships and theorems of parallelograms.</p> <hr/> <p><b>Access Point</b></p> <p>MA.912.GR.1.AP.4 Use the relationships and theorems about parallelograms. Solve mathematical and/or real-world problems involving postulates, relationships and theorems of parallelograms.</p> <hr/> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: quadrilateral, parallelogram, opposite sides, parallel, congruent, consecutive angles, supplementary angles, bisect, diagonals, congruent triangles, polygons.</li> <li>• Understand that parallel lines are lines that do not cross and are the same distance from each other.</li> <li>• Understand that a quadrilateral is a polygon which has four sides and four angles.</li> <li>• Understand that a parallelogram is a special type of quadrilateral.</li> <li>• Understand that a parallelogram has opposite sides that are parallel.</li> <li>• Understand that the opposite angles of a parallelogram are congruent.</li> <li>• Understand that consecutive angles of a parallelogram are supplementary.</li> <li>• Understand that diagonals bisect each other.</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that diagonals of a parallelogram form two congruent triangles.</li> </ul>
<p>MA.912.GR.1.5</p>	<p>Prove relationships and theorems about trapezoids. Solve mathematical and real-world problems involving postulates, relationships and theorems of trapezoids.</p> <p><b>Access Point</b></p> <p>MA.912. GR.1. AP.5 Use the relationships and theorems about trapezoids. Solve mathematical and/or real-world problems involving postulates, relationships and theorems of trapezoids.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: trapezoid, parallel lines, polygon, quadrilateral, isosceles trapezoid, congruent, supplementary angles, diagonals, opposite angles, base angles.</li> <li>• Understand that parallel lines are lines that do not cross and are the same distance from each other.</li> <li>• Understand that a quadrilateral is a polygon which has four sides and four angles.</li> <li>• Understand that a trapezoid is a special type of quadrilateral.</li> <li>• Understand that a trapezoid consists of at least one pair of parallel sides.</li> <li>• Understand that a trapezoid can be an isosceles trapezoid (legs are the same length)</li> <li>• Understand that in an isosceles trapezoid the diagonals are congruent.</li> <li>• Understand in an isosceles trapezoid the opposite angles are supplementary.</li> <li>• Understand that in an isosceles trapezoid the base angles are congruent</li> </ul>
<p>MA.912.GR.1.6</p>	<p>Solve mathematical and real-world problems involving congruence or similarity in two-dimensional figures.</p> <p><b>Access Point</b></p> <p>MA.912.GR.1.AP.6 Use the definitions of congruent or similar figures to solve mathematical and/or real-world</p>

	<p>problems involving two-dimensional figures.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: two-dimensional figures, congruent, similar, proportion, angles, multiplication, cross multiplication, fractions.</li> <li>• Understand that tools can be used to show that sides and angles are congruent (ruler, gridded paper, protractor, etc.)</li> <li>• Understand basic multiplication of two numbers.</li> <li>• Understand that cross multiplication can be used to show that two fractions are proportional.</li> <li>• Understand that two congruent two-dimensional figures are figures that are the same shape and size.</li> <li>• Understand that two similar two-dimensional figures are two figures whose sides are in proportion to each other, and their angles are congruent.</li> </ul>
<p><b><i>MA.912.GR.2 Apply properties of transformations to describe congruence or similarity.</i></b></p>	
<p>MA.912.GR.2.1</p>	<p>Given a preimage and image, describe the transformation and represent the transformation algebraically using coordinates.</p> <p><b>Access Point</b></p> <p>MA.912.GR.2.AP.1a Given a preimage and image, identify the transformation.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: transformation, pre-image, image, slide, flip, rotate, rotation, reflection, translation, line, axis, point, figure, up, down, right, left</li> <li>• Understand that a rotation, reflection and translation is a type of a transformation</li> <li>• Understand that the pre-image is the figure before the transformation. The image is the figure after the transformation</li> </ul>

- Understand that a translation is a slide of every point in the figure the same distance and the same direction (up, down, right, or left).
- Understand that a reflection is a flip of every point in a figure over a line, axis, point, etc.
- Understand that a rotation is a turn of every point in a figure about a point.

MA.912.GR.2.AP.1b Select the algebraic coordinates that represent the transformation.

**Essential Understandings:**

- Understand the following terms and vocabulary: transformation, pre-image, image, slide, flip, rotate, rotation, reflection, translation, line, axis, point, figure, up, down, right, left, signs, opposite signs, x-axis, y-axis, x-coordinate, y-coordinate, addition, subtraction, counterclockwise, coordinate value, positive numbers, negative numbers.
- Understand basic addition and subtraction.
- Understand that negative numbers and positive numbers have opposite signs.
- Understand that a rotation, reflection, and translation is a type of a transformation.
- Understand that the pre-image is the figure before the transformation. The image is the figure after the transformation.
- Understand that a translation is a slide of every point in the figure the same distance and the same direction (up, down, right, or left)
- Understand that a reflection is a flip of every point in a figure over a line, axis, point, etc.
- Understand that a rotation is a turn of every point in a figure about a point.
- Understand to reflect a point over the x-axis the y-coordinate changes signs.

For example  $(2,1)$  reflected over the x-axis becomes  $(2,-1)$

	<ul style="list-style-type: none"> <li>• Understand to reflect a point over the y-axis the x- coordinate changes signs.</li> </ul> <p>For example (2,1) reflected over coordinate the y-axis becomes (-2,1)</p> <ul style="list-style-type: none"> <li>• Understand to translate (slide) a point right or left, a value will be added/subtracted to the x-coordinate.</li> </ul> <p>For example, (2,1) translated to the right 6 units (8,1)</p> <ul style="list-style-type: none"> <li>• Understand to translate (slide) a point up or down, a value will be added/subtracted to the y-coordinate.</li> </ul> <p>For example, (2,1) translated up 6 units (2,7)</p> <ul style="list-style-type: none"> <li>• Understand to rotate a point counterclockwise 90 degrees centered at the origin, the coordinate point will change from (x,y) to (-y,x).</li> </ul> <p>For example, (2,1) rotated counterclockwise 90 degrees becomes (-1,2)</p>
MA.912.GR.2.2	<p>Identify transformations that do or do not preserve distance.</p> <p><b>Access Point</b></p> <p>MA.912.GR.2.AP.2 Select a transformation that preserves distance.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: preserve distance, transformations, translation, reflection, rotation, congruent, figure, point, slide, up, down, right, left, flip, line, axis</li> <li>• Understand the following transformations preserve distance: translation, reflection, rotation</li> <li>• Understand that preserve distance means the distance between the points of the figure will remain the same when a figure is transformed (The figures are congruent)</li> <li>• Understand that a translation is a slide of</li> </ul>

	<p>every point in the figure the same distance and the same direction (up, down, right, or left)</p> <ul style="list-style-type: none"> <li>• Understand that a reflection is a flip of every point in a figure over a line, axis, point, etc.</li> <li>• Understand that a rotation is a turn of every point in a figure about a point</li> </ul>
<p>MA.912.GR.2.3</p>	<p>Identify a sequence of transformations that will map a given figure onto itself or onto another congruent or similar figure.</p> <p><b>Access Point</b></p> <p>MA.912.GR.2.AP.3 Identify a given sequence of transformations, that includes translations or reflections, that will map a given figure onto itself or onto another congruent figure.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: transformation, translation, reflection, slide, figure, up, down, right, left, line, axis, point, x-axis, y-axis, add, subtract, value, y-coordinate, x-coordinate, sequence, mapping a figure.</li> <li>• Understand that a translation is a slide of every point in the figure the same distance and the same direction (up, down, right, or left)</li> <li>• Understand that a reflection is a flip of every point in a figure over a line, axis, point, etc.</li> <li>• Understand to reflect a point over the x-axis the y-coordinate changes signs.</li> </ul> <p>For example, (2,1) reflected over the x-axis becomes (2,-1)</p> <ul style="list-style-type: none"> <li>• Understand to reflect a point over the y-axis the x- coordinate changes signs.</li> </ul> <p>For example, (2,1) reflected over coordinate the y-axis becomes (-2,1)</p> <ul style="list-style-type: none"> <li>• Understand to translate (slide) a point right or left, a value will be added/subtracted to</li> </ul>



	<p>the x-coordinate.</p> <p>For example, (2,1) translated to the right 6 units (8,1)</p> <ul style="list-style-type: none"> <li>• Understand to translate (slide) a point up or down, a value will be added/subtracted to the y-coordinate.</li> </ul> <p>For example, (2,1) translated up 6 units (2,7)</p> <ul style="list-style-type: none"> <li>• Understand that a sequence of transformations can be used to move a figure and then move it back on itself.</li> </ul> <p>This is called mapping a figure on itself. For example, if a figure was shifted to the right and reflected over the x-axis, what steps would be needed to return the figure to its original location?</p> <ul style="list-style-type: none"> <li>• Understand that a sequence of transformations can be used to move a figure on top of a congruent figure.</li> </ul> <p>This is called mapping a figure onto another figure</p>
<p>MA.912.GR.2.5</p>	<p>Given a geometric figure and a sequence of transformations, draw the transformed figure on a coordinate plane.</p> <p><b>Access Point</b></p> <p>MA.912.GR.2.AP.5 Given a geometric figure and a sequence of transformations, select the transformed figure on a coordinate plane.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: transformation, translation, reflection, rotation, slide, figure, up, down, right, left, line, axis, point, x-axis, y-axis, add, subtract, value, y-coordinate, x-coordinate, sequence, mapping a figure, coordinate plane.</li> <li>• Understand that a translation is a slide of every point in the figure the same distance and the same direction (up, down, right, or left)</li> <li>• Understand that a reflection is a flip of every point in a figure over a line, axis, point, etc.</li> </ul>

- Understand that a rotation is a turn of every point in a figure about a point.
- Understand to reflect a point over the x-axis the y-coordinate changes signs.

For example  $(2,1)$  reflected over the x-axis becomes  $(2,-1)$

- Understand to reflect a point over the y-axis the x- coordinate changes signs.

For example  $(2,1)$  reflected over coordinate the y-axis becomes  $(-2,1)$

- Understand to translate (slide) a point right or left, a value will be added/subtracted to the x-coordinate.

For example,  $(2,1)$  translated to the right 6 units  $(8,1)$ .

- Understand to translate (slide) a point up or down, a value will be added/subtracted to the y-coordinate.

For example,  $(2,1)$  translated up 6 units  $(2,7)$

- Understand to rotate a point counterclockwise 90 degrees centered at the origin, the coordinate point will change from  $(x,y)$  to  $(-y,x)$

For example,  $(2,1)$  rotated counterclockwise 90 degrees becomes  $(-1,2)$

- Understand that given a figure and a sequence of transformations (rotation, translations, and reflection) identify where the figure is mapped on a coordinate plane (new location)

MA.912.GR.2.6

Apply rigid transformations to map one figure onto

another to justify that the two figures are congruent.

**Access Point**

MA.912.GR.2.AP.6 Use rigid transformations that includes translations or reflections to map one figure onto another to show that the two figures are congruent.

**Essential Understandings:**

- Understand the following terms and conditions: transformation, translation, reflection, slide, figure, up, down, right, left, line, axis, point, x-axis, y-axis, add, subtract, value, y-coordinate, x-coordinate, sequence, mapping a figure, congruent, two-dimensional.
- Understand that tools can be used to show that sides and angles are congruent (ruler, gridded paper, protractor, etc.)
- Understand that a translation is a slide of every point in the figure the same distance and the same direction (up, down, right, or left)
- Understand that a reflection is a flip of every point in a figure over a line, axis, point, etc.
- Understand to reflect a point over the x-axis the y-coordinate changes signs.

For example  $(2,1)$  reflected over the x-axis becomes  $(2,-1)$

- Understand to reflect a point over the y-axis the x- coordinate changes signs.

For example  $(2,1)$  reflected over coordinate the y-axis becomes  $(-2,1)$

- Understand to translate (slide) a point right or left, a value will be added/subtracted to the x-coordinate.

For example,  $(2,1)$  translated to the right 6 units  $(8,1)$

- Understand to translate (slide) a point up or down, a value will be added/subtracted to

	<p>the y-coordinate.</p> <p>For example, (2,1) translated up 6 units (2,7)</p> <ul style="list-style-type: none"> <li>• Understand that two congruent two-dimensional figures are figures that are the same shape and size.</li> <li>• Understand that a sequence of transformations can be used to move a figure on top of a congruent figure.</li> </ul> <p>This is called mapping a figure onto another figure</p>
<p>MA.912.GR.2.8</p>	<p>Apply an appropriate transformation to map one figure onto another to justify that the two figures are similar.</p> <p><b>Access Point</b></p> <p>MA.912.GR.2.AP.8 Identify an appropriate transformation to map one figure onto another to show that the two figures are similar.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and conditions: transformation, translation, reflection, rotation, slide, figure, up, down, right, left, line, axis, point, x-axis, y-axis, add, subtract, value, y-coordinate, x-coordinate, sequence, mapping a figure, congruent angles, similar, proportion, counterclockwise, two-dimensional, fractions, multiplication, cross multiplication.</li> <li>• Understand that tools can be used to show that sides and angles are congruent (ruler, gridded paper, protractor, etc.)</li> <li>• Understand basic multiplication of two numbers.</li> <li>• Understand that a translation is a slide of every point in the figure the same distance and the same direction (up, down, right, or left).</li> <li>• Understand that a reflection is a flip of every point in a figure over a line, axis, point, etc.</li> <li>• Understand that a rotation is a turn of every point in a figure about a point.</li> </ul>

- Understand to reflect a point over the x-axis the y-coordinate changes signs.

For example  $(2,1)$  reflected over the x-axis becomes  $(2,-1)$ .

- Understand to reflect a point over the y-axis the x- coordinate changes signs.

For example  $(2,1)$  reflected over coordinate the y-axis becomes  $(-2,1)$ .

- Understand to translate (slide) a point right or left, a value will be added/subtracted to the x-coordinate.

For example,  $(2,1)$  translated to the right 6 units  $(8,1)$ .

- Understand to translate (slide) a point up or down, a value will be added/subtracted to the y-coordinate.

For example,  $(2,1)$  translated up 6 units  $(2,7)$

- Understand to rotate a point counterclockwise 90 degrees centered at the origin, the coordinate point will change from  $(x,y)$  to  $(-y,x)$

For example,  $(2,1)$  rotated counterclockwise 90 degrees becomes  $(-1,2)$

- Understand that two similar two-dimensional figures are two figures whose sides are in proportion to each other, and their angles are congruent.
- Understand that cross multiplication can be used to show that two fractions are proportional.
- Understand that a sequence of

transformations can be used to move a figure on top of another figure\

- When the figures have congruent angles, and the sides are in proportion to each other they are similar.

This is called mapping a figure onto another figure

**MA.912.GR.3 Use coordinate geometry to solve problems or prove relationships.**

MA.912.GR.3.1

Determine the weighted average of two or more points on a line.

**Access Point**

MA.912.GR.3.AP.1 Select the weighted average of two or more points on a line.

**Essential Understandings:**

- Understand the following terms and vocabulary: ratio, graph, point, line segment, x-variable, y-variable, x-axis, y-axis, weighted average, integer, partitions,
- Understand addition and subtraction of integers.
- Understand using basic ratios and how they are represented.
- Understand when given two points and a ratio, a weighted average will be used to locate a third point.

Ex.

$$A = (2,5)$$

$$B = (-2,1)$$

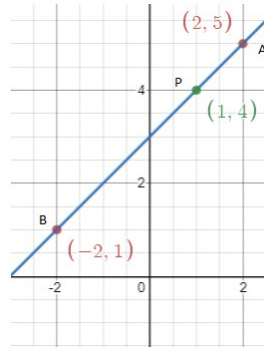
Ratio 1:3

Where should point P be located so that it partitions Segment AB into a 1:3 ratio?

Start with point A (Using segment AB)

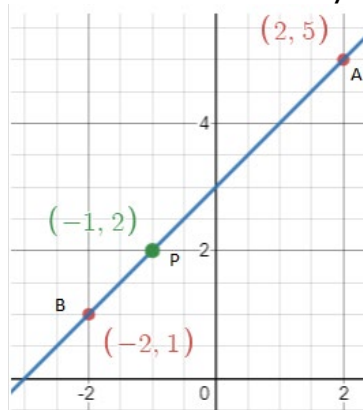
For the x-variable 1 is one away from 2 and three away from -2

For the y-variable 4 is one away from 5 and three away from 1



Where should point P be located so that it partitions Segment BA into a 1:3 ratio?

Start with B (using segment BA)  
 For the x-variable -1 is one away from -2 and three away from 2  
 For the y-variable 2 is one away from 1 and three away from 5



MA.912.GR.3.2

Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals

**Access Point**

MA.912.GR.3.AP.2 Use coordinate geometry to classify definitions, properties and theorems involving circles, triangles, or quadrilaterals.

**Essential Understandings:**

- Understand the following terms and vocabulary: triangle, polygon, sides, equilateral triangle, isosceles triangle, scalene triangle, congruent, center, circle, length, parallel, parallelogram, square,

rectangle, trapezoid, opposite, rhombus, diameter, radius, distance formula, center point.

- Understand that a triangle is a polygon which consists of three sides.
- Understand that triangles can be named by their sides: equilateral, isosceles, scalene.
- Understand that the distance formula ( $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ) can be used to find the length of a side of a polygon.
- Understand that the distance formula ( $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ) can be used to identify which type of triangle is formed:

Equilateral – all three sides have the same length.

Isosceles – two sides have the same length.

Scalene – none of the sides have the same length.

- Understand that a quadrilateral is a polygon which has four sides.
- Understand that the following quadrilaterals have parallel sides: Square, Rectangle, parallelogram, trapezoid.
- Understand to determine if two sides are parallel, the slope formula is used.
- Understand that two sides are parallel, if their slopes are the same.
- Understand that the following quadrilaterals have opposite sides that are congruent: square, parallelogram, rectangle.
- Understand the following quadrilaterals have four sides that are congruent: square, rhombus.
- Understand to prove that sides of a quadrilateral are congruent, the distance formula ( $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ) is used.
- Understand that a circle consists of all the points on a given plane the same distance from a center point.
- Understand the distance from the side of a



	<p>circle to the center point is called the radius.</p> <ul style="list-style-type: none"> <li>• Understand the distance across the circle going through the center point is called the diameter.</li> </ul>
<p>MA.912.GR.3.3</p>	<p>Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.</p> <p><b>Access Point</b></p> <p>MA.912.GR.3.AP.3 Use coordinate geometry to solve mathematical geometric problems involving lines, triangles and quadrilaterals.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: triangle, polygon, sides, equilateral triangle, isosceles triangle, scalene triangle, congruent, length, parallel, parallelogram, square, rectangle, trapezoid, opposite, rhombus, distance formula, line, line segment, infinite.</li> <li>• Understand that a triangle is a polygon which consists of three sides.</li> <li>• Understand that triangles can be named by their sides: equilateral, isosceles, scalene.</li> <li>• Understand that the distance formula (<math>d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}</math>) or counting can be used to find the length of a side of a polygon.</li> <li>• Understand that the distance formula or counting can be used to identify which type of triangle is formed: <ul style="list-style-type: none"> <li>Equilateral – all three sides have the same length.</li> <li>Isosceles – two sides have the same length.</li> <li>Scalene – none of the sides have the same length.</li> </ul> </li> <li>• Understand that a quadrilateral is a polygon which has four sides.</li> <li>• Understand that the following quadrilaterals have parallel sides: Square, Rectangle,</li> </ul>

	<p>parallelogram, trapezoid.</p> <ul style="list-style-type: none"> <li>• Understand to determine if two sides are parallel, the slope formula is used.</li> <li>• Understand that two sides are parallel, if their slopes are the same.</li> <li>• Understand that the following quadrilaterals have opposite sides that are congruent: square, parallelogram, rectangle.</li> <li>• Understand the following quadrilaterals have four sides that are congruent: square, rhombus.</li> <li>• Understand to prove that sides of a quadrilateral are congruent, the distance formula or counting is used.</li> <li>• Understand that the slope of a line can be determined by using the slope formula and two points on the line.</li> <li>• Understand that once the slope is determined, the equation of a line can be determined using the slope and a point on the line.</li> <li>• Understand that the distance of a line cannot be determined as it is infinite.</li> <li>• Understand that the distance of a line segment (portion of a line) can be determined by using the distance formula.</li> </ul>
MA.912.GR.3.4	<p>Use coordinate geometry to solve mathematical and real-world problems on the coordinate plane involving perimeter or area of polygons.</p> <p><b>Access Point</b></p> <p>MA.912.GR.3.AP.4 Solve mathematical and/or real-world problems on the coordinate plane involving perimeter or area of a three- or four-sided polygon.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: distance formula, triangle, base, height, polygon, length, perimeter, area, square, rectangle, parallelogram, trapezoid four-sided figure, three-sided figure.</li> <li>• Understand the lengths of the sides of a polygon are determined by counting or using</li> </ul>

the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Understand the area of a triangle (three-sided figure), when given the length of the base and the height, is calculated with the following formula:  $A = \frac{1}{2}bh$ . (b equals the length of the base and h equals the height).
- Understand the perimeter of a triangle is determined by adding the length of all three sides.
- Understand that the perimeter of a four-sided figure is determined by adding the lengths of all four sides.
- Understand the area of following four-sided figures can be determined using the following formulas:

Square, rectangle, parallelogram:  $A = bh$

Trapezoid:  $A = \frac{1}{2}(b_1 + b_2)h$

**MA.912.GR.4 Use geometric measurement and dimensions to solve problems.**

MA.912.GR.4.1

Identify the shapes of two-dimensional cross sections of three-dimensional figures.

**Access Point**

MA.912.GR.4.AP.1 Identify the shape of a two-dimensional cross section of a three-dimensional figure.

**Essential Understandings:**

- Understand the following terms and vocabulary: two-dimensional figure, three-dimensional figure, cross section, square, circle, triangle, rectangle, cube, cylinder, cone, pyramid, length, width, height, parallel, base, shape, solid.
- Understand that a two-dimensional figure has two dimensions, width, and height, and lies in one plane.

ex. Circle, square, triangle, rectangle, etc.

	<ul style="list-style-type: none"> <li>• Understand that a three-dimensional figure has three dimensions, Length, width, and height.  ex. Cube, cylinder, cone, pyramid, etc.</li> <li>• Understand that a cross section is a shape made when a solid is cut through parallel to the base.  ex. Cutting through a cube parallel to its base, the cross section is a square</li> </ul>
MA.912.GR.4.2	<p>Identify three-dimensional objects generated by rotations of two-dimensional figures.</p> <p><b>Access Point</b> MA.912.GR.4.AP.2 Identify a three-dimensional object generated by the rotation of a two-dimensional figure.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: two-dimensional figure, three-dimensional figure, width, height, length, plane, circle, square, triangle, rectangle, cube, cylinder, cone, pyramid, rotated, generate new figure, line.</li> <li>• Understand that a two-dimensional figure has two dimensions, width, and height, and lies in one plane.  Ex. Circle, square, triangle, rectangle, etc.</li> <li>• Understand that a three-dimensional figure has three dimensions, Length, width, and height.  Ex. Cube, cylinder, cone, pyramid, etc.</li> <li>• Understand when a two-dimensional figure is rotated, a three-dimensional figure is generated.  Ex. Rotating a triangle around a line</li> </ul>

	<p>generates a cone. Rotating a rectangle around a line generates a cylinder</p>
<p>MA.912.GR.4.3</p>	<p>Extend previous understanding of scale drawings and scale factors to determine how dilations affect the area of two-dimensional figures and the surface area or volume of three-dimensional figures.</p> <p><b>Access Point</b></p> <p>MA.912.GR.4.AP.3 Select the effect of a dilation on the area of two-dimensional figures and/or surface area or volume of three-dimensional figures.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: two-dimensional figure, three-dimensional figure, area, surface area, volume, dilation, image, width, length, height, plane, circle, square, triangle, rectangle, cube, cylinder, cone, pyramid, exponents.</li> <li>• Understand that a two-dimensional figure has two dimensions, width, and height, and lies in one plane.  Ex. Circle, square, triangle, rectangle, etc.</li> <li>• Understand that a three-dimensional figure has three dimensions, Length, width, and height.  Ex. Cube, cylinder, cone, pyramid, etc.</li> <li>• Understand how to use exponents (square and cube only)</li> <li>• Understand how to calculate the area of a two-dimensional figure.</li> <li>• Understand how to calculate the surface area of a three-dimensional figure.</li> <li>• Understand how to calculate the volume of a three-dimensional figure.</li> <li>• Understand how dilation affects area, surface area, and volume.</li> </ul>

Ex:

The dilation ( $a$ ) effects area of a two-dimensional figure by creating a new image whose area is  $a^2$  times larger.

The dilation ( $a$ ) effects surface area of a three-dimensional figure by creating a new image whose surface area is  $a^2$  times larger.

The dilation ( $a$ ) effects volume of a three-dimensional figure by creating a new image whose area is  $a^3$  times larger.

Given a 2 x 3 rectangle.

Area = 6 square units

Dilation	Length	Width	Area	Effect
2 times larger	4	6	24	$2^2 = 4$ Area is 4 times larger
3 times larger	6	9	54	$3^2 = 9$ Area is 9 times larger

Given a 2 x 3 x 2 rectangular solid.

Surface area = 32 square units

Volume = 12 cubic units

Dilation	Length	Width	height	Surface Area	Effect on Surface Area	Volume	Effect on volume
2 times larger	4	6	4	128	$2^2 = 4$ Surface Area is 4 times larger	96	$2^3 = 8$ Volume is 8 times larger
3 times larger	6	9	6	288	$3^2 = 9$ Surface Area is 9 times larger	324	$3^3 = 27$ Volume is 27 times larger

MA.912.GR.4.4

Solve mathematical and real-world problems involving the area of two-dimensional figures.

**Access Point**

MA.912.GR.4.AP.4 Solve mathematical and/or real-world problems involving the area of triangles, squares, circles or rectangles.

**Essential Understandings:**

- Understand the following terms and vocabulary: triangle, square, circle, rectangle, multiply ( $\times$ ), two-dimensional, side, angles, parallel, right angle, center, radius, diameter,  $\pi$ , infinite, point, base, height, length, opposite sides.
- Understand how to multiply two numbers together.
- Understand how to take half of a number.
- Understand that a triangle is a two-dimensional figure with three sides and three angles.

- Understand that a square is a two-dimensional figure with four sides and four right angles where all the sides are the same length and opposite sides are parallel.
- Understand that a rectangle is a two-dimensional figure with four sides and four right angles where the sides opposite each other are the same length and parallel.
- Understand that a circle is a two-dimensional figure made up of an infinite number of points the same distance from the center.
- Understand that the radius of a circle is the distance from the center to the sides.
- Understand that the radius is half the diameter (distance across the center of the circle).
- Understand the following area formulas:

Square and rectangle -  $A = bh$  (b = length of base, h = height of the figure).

Triangle -  $A = \frac{1}{2}bh$  (b = length of base, h = height of the figure).

Circle -  $A = \pi r^2$  (r = radius of the circle,  $\pi \approx 3.14$ ).

MA.912.GR.4.5

Solve mathematical and real-world problems involving the volume of three-dimensional figures limited to cylinders, pyramids, prisms, cones and spheres.

**Access Point**

MA.912.GR.4.AP.5 Solve mathematical or real-world problems involving the volume of three-dimensional figures limited to cylinders, pyramids, prisms, or cones.

**Essential Understandings:**

- Understand the following terms and vocabulary: cylinders, pyramids, prisms, cones, three-dimensional figure, circular ends, parallel, curved side, rectangular base, triangular side, multiplication, radius, diameter, length, width, height, volume, circular base.
- Understand multiplication of three numbers

- Understand how to take a half and a third of a number.
- Understand that a cylinder is a three-dimensional figure is made up of two circular ends that are parallel to each other and are connected by a curved side (tube, soup can, etc.).
- Understand that the radius of a circle is the distance from the center to the sides.
- Understand that the radius is half the diameter (distance across the center of the circle).
- Understand that a pyramid can be a three-dimensional figure with a rectangular base and four triangular sides.
- Understand that a prism can be a three-dimensional figure with identical parallel ends (ex. rectangle, square, triangle) and multiple rectangular sides (ex. if the base is a triangle, the prism will have three rectangular sides).
- Understand that a cone is a three-dimensional figure with a circular base, a point at the opposite end and a curved side .
- Understand the following volume formulas:

Cylinder -  $V = \pi r^2 h$  ( $\pi \approx 3.14$ ,  $r$  = radius,  $h$  = height)

Cone -  $V = \frac{1}{3} \pi r^2 h$  ( $\pi \approx 3.14$ ,  $r$  = radius,  $h$  = height)

Rectangular Prism -  $V = lwh$  ( $l$  = length,  $w$  = width,  $h$  = height)

Triangular Prism -  $V = \frac{1}{2} lwh$  ( $l$  = length,  $w$  = width,  $h$  = height)

Pyramid -  $V = \frac{1}{3} lwh$  ( $l$  = length,  $w$  = width,  $h$  = height)

MA.912.GR.4.6

Solve mathematical and real-world problems involving the surface area of three-dimensional figures limited to cylinders, pyramids, prisms, cones and spheres.

**Access Point**

MA.912.GR.4.AP.6 Solve mathematical or real-world



problems involving the surface area of three-dimensional figures limited to cylinders, pyramids, prisms, and cones.

**Essential Understandings:**

- Understand the following terms and vocabulary: cylinders, pyramids, rectangular prisms, triangular prisms, cones, three-dimensional figure, circular ends, parallel, curved side, rectangular base, triangular side, multiplication, radius, diameter, length, width, height, surface area, circular base, slant height, perimeter, center.
- Understand multiplication of two numbers
- Understand how to take a half of a number.
- Understand that a cylinder is a three-dimensional figure is made up of two circular ends that are parallel to each other and are connected by a curved side (tube, soup can, etc.).
- Understand that the radius of a circle is the distance from the center to the sides.
- Understand that the radius is half the diameter (distance across the center of the circle).
- Understand that a pyramid can be a three-dimensional figure with a rectangular base and four triangular sides.
- Understand that a prism can be a three-dimensional figure with identical parallel ends (ex. rectangle, square, triangle) and multiple rectangular sides (ex. if the base is a triangle, the prism will have three rectangular sides).
- Understand that a cone is a three-dimensional figure with a circular base, a point at the opposite end and a curved side.
- Understand the following surface area formulas:

Cylinder -  $SA = 2\pi r^2 + 2\pi rh$  ( $r =$  radius,  $h =$  height,  $\pi \approx 3.14$ )

Cone -  $SA = \pi rs + \pi r^2$  ( $r =$  radius,  $s =$

slant height,  $\pi \approx 3.14$ )

Rectangular Prism -  $SA = 2(lw + lh + wh)$  ( $l$  = length,  $w$  = width,  $h$  = height)

Pyramid -  $SA = B + \frac{1}{2}ps$  ( $B$  = area of the base,  $p$  = perimeter of the base,  $s$  = slant height)

**MA.912.GR.5 Make formal geometric constructions with a variety of tools and methods.**

MA.912.GR.5.1

Construct a copy of a segment or an angle.

**Access Point**

MA.912.GR.5.AP.1 Construct a copy of a segment.

**Essential Understandings:**

- Understand the following terms and vocabulary: segment, reference line and endpoint, straightedge, portion, compass, line, span, point.
- Understand that a segment is a portion of a line.

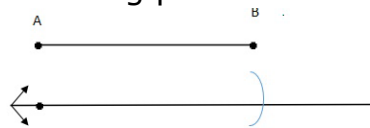
It begins at one point on the line and ends at another point.

These points are known as the endpoints of the segment.

- Understand to copy a segment the following steps need to be followed:

Draw a line with a straightedge.

Place a starting point on the line.

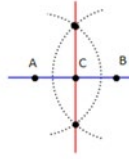


Place the point of the compass on point  $A$

Stretch the compass so that the pencil is exactly on  $B$ .

Without changing the span of the compass, place the compass point on the starting point on the reference line

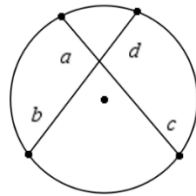
	<p>and swing the pencil so that it crosses the reference line. Label the new line segment</p>
<p>MA.912.GR.5.2</p>	<p>Construct the bisector of a segment or an angle, including the perpendicular bisector of a line segment.</p> <p><b>Access Point</b></p> <p>MA.912.GR.5.AP.2 Construct the bisector of a segment, including the perpendicular bisector of a line segment.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: line segment, point, arc, intersect, perpendicular, bisect, congruent, midpoint, compass, span, arc, straightedge.</li> <li>• Understand that bisect means to divide the segment into two equal parts.</li> <li>• Understand that a perpendicular bisector is a perpendicular line or segment that passes through the midpoint of a line.</li> <li>• Understand to bisect a segment the following steps need to be followed:</li> </ul> <p style="padding-left: 40px;">Place compass point on <math>A</math> and stretch the compass more than halfway to point <math>B</math>, but not beyond <math>B</math>. With this length, swing a large arc that will go both above and below <math>\overline{AB}</math> Without changing the span on the compass, place the compass point on <math>B</math> and swing the arc again. The two arcs that have been created should intersect. With a straightedge, connect the two points of intersection. This new straight line bisects <math>\overline{AB}</math> Label the point where the new line and <math>\overline{AB}</math> cross as <math>C</math> <math>\overline{AB}</math> has now been bisected and <math>AC = CB</math> (It could also be said that the segments are congruent, <math>\overline{AC} \cong \overline{CB}</math>)</p>



MA.912.GR.5.3	Construct the inscribed and circumscribed circles of a triangle.
	<p><b>Access Point</b></p> <p>MA.912.GR.5.AP.3 Select the inscribed and circumscribed circles of a triangle.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: inscribed circle, circumscribed circle, triangle, sides, circle, vertices.</li> <li>• Understand that inscribed circle is a circle inside a triangle that touches all three sides.</li> <li>• Understand that a circumscribed circle is a circle that is on the outside of a triangle touching all three vertices (points of the triangle)</li> </ul>
<b>MA.912.GR.6 Use properties and theorems related to circles.</b>	
MA.912.GR.6.1	Solve mathematical and real-world problems involving the length of a secant, tangent, segment or chord in a given circle.
	<p><b>Access Point</b></p> <p>MA.912.GR.6.AP.1 Identify and describe the relationship involving the length of a secant, tangent, segment or chord in a given circle.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: tangent, secant, chord, segment, circle, line, length, intersecting, product, point, inside, outside, squared, equal.</li> <li>• Understand how to multiply two numbers.</li> <li>• Understand the secant of a circle is a line that touches the circle in two places.</li> <li>• Understand the tangent of a circle is a line that touches the circle in one place.</li> <li>• Understand the segment is part of a line.</li> </ul>

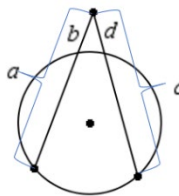
- Understand the chord of a circle is a segment that touches two points on a circle.
- Understand when two chords intersect, the product of the lengths of the segments of one chord equals the product of the lengths of segments of the intersecting chord (see image below)

$$ac = bd$$



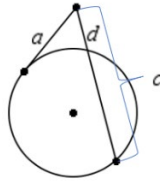
- Understand when two secants intersect the circle and meet at the same point outside the circle, the product of the length of the segment outside the circle and the length of the segment is equal to the product of the other length of the segment outside the circle and the length of the other segment (see image below)

$$ab = cd$$



- Understand when a secant and a tangent intersect outside the circle, the product of the length of the segment outside the circle and the length of the segment inside the circle equals the length of the tangent segment squared (see image below)

$$cb = a^2$$



MA.912.GR.6.2

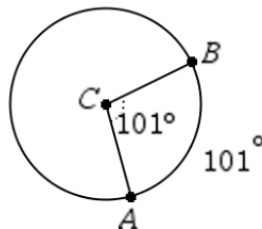
Solve mathematical and real-world problems involving the measures of arcs and related angles.

**Access Point**

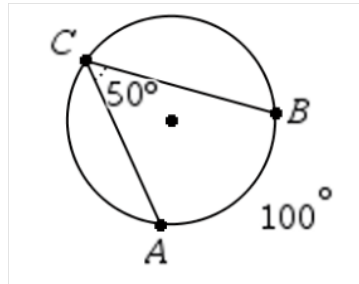
MA.912.GR.6.AP.2 Identify the relationship involving the measures of arcs and related angles, limited to central, inscribed and intersections of a chord, secants or tangents.

**Essential Understandings:**

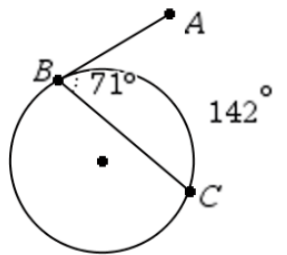
- Understand the following terms and vocabulary: tangent, secant, chord, segment, circle, line, length, intersecting, intercepting, vertex, arc, angle, central angle, radii, point, equal, inscribed angle, relationship.
- Understand how to multiply two numbers.
- Understand the secant of a circle is a line that touches the circle in two places.
- Understand the tangent of a circle is a line that touches the circle in one place.
- Understand the segment is part of a line.
- Understand the chord of a circle is a segment that touches two points on a circle.
- Understand the central angle is formed by two radii that meet at the center of the circle (vertex)
- Understand the central angle is the same measure as the intercepted arc. The relationship between the central angle and the intercepted arc is  $m\angle ACB = m\widehat{AB}$



- Understand an inscribed angle is the angle formed by two intersecting chords (secants) that intersect on the circle forming the vertex of the angle. The relationship between the vertex and the intercepted arc formed is  $m\angle ACB = 2(m\widehat{AB})$



- Understand when a chord and a tangent intersect on a circle, the vertex of the created angle is on the circle. The relationship between the created angle and the intercepted arc is  $m\angle ABC = 2(m\widehat{BC})$



MA.912.GR.6.3

Solve mathematical problems involving triangles and quadrilaterals inscribed in a circle.

**Access Point**

MA.912.GR.6.AP.3 Identify and describe the relationship involving triangles and quadrilaterals inscribed in a circle.

**Essential Understandings:**

- Understand the following terms and vocabulary: supplementary angles, semicircle, right angle, inscribe, degrees, quadrilateral, opposite angles, circle, addition sign.
- Understand how to add two numbers.
- Understand an angle inscribed in a semicircle is a right angle (Thales Theorem).

	<ul style="list-style-type: none"> <li>Understand when a quadrilateral is inscribed in a circle its opposite angles add up to 180 degrees (supplementary angles).</li> </ul>
MA.912.GR.6.4	<p>Solve mathematical and real-world problems involving the arc length and area of a sector in a given circle.</p> <p><b>Access Point</b></p> <p>MA.912.GR.6.AP.4 Identify and describe the relationship involving the arc length and area of a sector in a given circle.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>Understand the following terms and vocabulary: radius (<math>r</math>), arc length (<math>L</math>), angle measure (<math>\theta</math>), pi (<math>\pi</math>), circle, sector, area, chord, arc, length, point, distance, radians, degrees, radii, intercept, squared.</li> <li>Understand the sector of a circle is formed when two radii intercept an arc.</li> </ul> <p>The angle measure (<math>\theta</math>) is in radians: <math>A = \frac{1}{2}r^2\theta</math>  The angle measure (<math>\theta</math>) is in degrees: <math>A = \frac{\theta}{360^\circ}\pi r^2</math></p> <ul style="list-style-type: none"> <li>Understand that the arc length is the distance from one point on the arc to another point on the arc.</li> <li>Understand that the length of the arc is found by multiplying the angle measure by the radius.</li> </ul> <p>The angle measure (<math>\theta</math>) is in radians:  <math>L = \theta r</math>  The angle measure (<math>\theta</math>) is in degrees: <math>L = \theta \left(\frac{\pi}{180}\right)r</math></p>
<b>MA.912.GR.7 Apply geometric and algebraic representations of conic sections.</b>	
MA.912.GR.7.2	<p>Given a mathematical or real-world context, derive and create the equation of a circle using key features.</p> <p><b>Access Point</b></p> <p>MA.912.GR.7.AP.2 Create the equation of a circle when</p>



	<p>given the center and radius.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: radius, circle, equation, substitution, center, variable, point, distance, formula, graph.</li> <li>• Understand the center of a circle is at point <math>(h, k)</math> on a graph.</li> <li>• Understand that the radius is represented by the variable <math>r</math>.</li> <li>• Understand that the radius is the distance from the side of the circle to the center of the circle.</li> <li>• Understand to create the equation of a circle, the following formula is used: <math>(x - h)^2 + (y - k)^2 = r^2</math>.</li> <li>• Understand the center point <math>(h, k)</math> and the radius <math>(r)</math> are substituted into the formula.</li> </ul>
<p>MA.912.GR.7.3</p>	<p>Graph and solve mathematical and real-world problems that are modeled with an equation of a circle. Determine and interpret key features in terms of the context.</p> <p><b>Access Point</b></p> <p>MA.912.GR.7.AP.3 Given an equation of a circle, identify center and radius, and graph the circle.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: radius <math>(r)</math>, circle, equation, center <math>(h, k)</math>, variable, point, distance, formula, right, left, up, down, graph.</li> <li>• Understand the equation of a circle uses the following formula: <math>(x - h)^2 + (y - k)^2 = r^2</math>.</li> <li>• Understand the center of a circle is at point <math>(h, k)</math> on a graph.</li> <li>• Understand that the radius is represented by the variable <math>r</math>.</li> <li>• Understand that the radius is the distance from the side of the circle to the center of the circle.</li> <li>• Understand to graph a circle, graph the center point first <math>(h, k)</math>.</li> </ul>

- Understand to graph the radius (distance from the center to the side), count from the center down, up, left and right. (If the radius is 3, count left 3, down 3, up 3 and right 3 from the center). Connect the dots to form the circle.

## 9-12 Data Analysis and Probability Strand

### ***MA.912.DP.1 Summarize, represent and interpret categorical and numerical data with one and two variables.***

MA.912.DP.1.1

Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.

#### **Access Point**

MA.912.DP.1.AP.1a Given a set of data, select an appropriate table or graph to represent categorical data and whether it is univariate or bivariate.

#### **Essential Understandings:**

- Understand the following terms and vocabulary: univariate data, bivariate data, categorical data, attribute, characteristics, bar graph, circle graph, frequency table, two-way table.
- Understand that categorical data is data that is classified by attributes or characteristics (Ex. Favorite color, type of car, number on a sports jersey).
- Understand that univariate data has a single characteristic or attribute (Ex. Favorite color is a single attribute).
- Understand that bivariate data has two characteristics or attributes (Ex. Height and weight).
- Understand that categorical data can be represented by the following graphs: circle graph, bar graph (single bar graph, double bar graph, stacked bar graph).
- Understand that categorical data can be represented by the following tables: frequency table (univariate data), or two-

	<p>way table (bivariate data).</p>
	<p>MA.912.DP.1.AP.1b Given a set of data, select an appropriate table or graph to represent numerical data and whether it is univariate or bivariate.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following term and vocabulary: numerical data, univariate data, bivariate data, variable, dot plots, scatter plots, stem plots, frequency table, two-way table, value, measure.</li> <li>• Understand that numerical data is data that can be measured (Ex. The number of people who like the color green).</li> <li>• Understand that univariate data has a single variable (Ex. Variable is type of car and the data is how many people own each type of car).</li> <li>• Understand that bivariate data is two numerical values paired with each other (Ex. Ordered pair (-2,3)).</li> <li>• Understand that numerical data can be represented by the following graphs: dot plots, scatter plots, or stem plots.</li> <li>• Understand that numerical data can be represented by the following tables: frequency table (univariate data), or two-way table (bivariate data)</li> </ul>
<p>MA.912.DP.1.2</p>	<p>Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.</p> <p><b>Access Point</b></p> <p>MA.912.DP.1.AP.2 Given a univariate or bivariate data distribution (numerical or categorical), identify the different components and quantities in the display.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following term and vocabulary: univariate distribution, bivariate distribution, attributes, numerical data, categorical data, characteristics, frequency</li> </ul>

	<p>table, two-way table joint frequency, marginal frequency, margins, measure.</p> <ul style="list-style-type: none"> <li>• Understand that categorical data is data that is classified by attributes or characteristics (Ex. Favorite color, type of car, number on a sports jersey)</li> <li>• Understand that numerical data is data that can be measured (Ex. The number of people who like the color green).</li> <li>• Understand that a bivariate distribution can be represented by a two-way table.</li> <li>• Understand that a univariate distribution can be represented by a frequency table.</li> <li>• Understand that a two-way table has two types of frequencies: joint frequencies (numbers inside the two-way table) and marginal frequencies (totals in the margins or edge of table).</li> </ul>
<p>MA.912.DP.1.3</p>	<p>Explain the difference between correlation and causation in the contexts of both numerical and categorical data.</p> <p><b>Access Point</b></p> <p>MA.912.DP.1.AP.3 Identify whether the data is explained by correlation or causation in the contexts of both numerical and categorical data.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: linear model, correlation coefficient, linear relationship, linear fit, correlation, causation, strength, data, fits a line, correlation coefficient (r), Linear pattern, linear relationship, categorical data, numerical data, attributes, characteristics, measure, experiment.</li> <li>• Understand that categorical data is data that is classified by attributes or characteristics (Ex. Favorite color, type of car, number on a sports jersey)</li> <li>• Understand that numerical data is data that can be measured (Ex. The number of people who like the color green.)</li> <li>• Understand the correlation measures the</li> </ul>

	<p>strength, the data, fits a line (linear pattern).</p> <ul style="list-style-type: none"> <li>• Understand that “r” represents the correlation coefficient.</li> <li>• Understand that the closer “r” is to -1 or 1, the stronger the data fits a linear relationship between x and y</li> <li>• Understand that the closer “r” is to 0 the weaker the data fits a linear relationship between x and y.</li> <li>• Understand that correlation does not prove causation [Ex. There is a strong linear relationship (correlation) between shoe size and reading levels. However, that does not mean that shoe size causes reading levels to increase.]</li> <li>• Understand that causation can only be proved with a well- designed experiment.</li> </ul>
<p>MA.912.DP.1.4</p>	<p>Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.</p> <p><b>Access Point</b></p> <p>MA.912.DP.1.AP.4 Given the mean or percentage and the margin of error from a sample survey, identify a population total.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: margin of error, level accuracy, experiment, range, trustworthiness, results, mean, percentage, interval, accuracy, population.</li> <li>• Understand that the margin of error describes the level of accuracy of an experiment. The margin of error describes a range that helps determine the trustworthiness of results (Ex: An election results poll has a +/- 3% margin of error.)</li> <li>• Understand that a combination of a mean or percentage with the margin of error gives us an interval that the population mean, or percentage may fall. (Ex. Mean = 10;</li> </ul>

	<p>margin of error = <math>\pm 2</math>; Interval = (8, 12) so the population mean may fall between 8 and 12).</p> <ul style="list-style-type: none"> <li>• Understand that a smaller margin of error indicates trustworthy results, and a larger margin of error means the results are not considered as accurate.</li> </ul>
<p><b>MA.912.DP.2 Solve problems involving univariate and bivariate numerical data.</b></p>	
<p>MA.912.DP.2.1</p>	<p>For two or more sets of numerical univariate data, calculate and compare the appropriate measures of center and measures of variability, accounting for possible effects of outliers. Interpret any notable features of the shape of the data distribution.</p> <p><b>Access Point</b>  MA.912.DP.2.AP.1 For two sets of numerical univariate data, calculate and compare the mean, median and range, then select the shape of the data from given graphs.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and Vocabulary: mean, range, numerical univariate data, graphs, median, less than, more than, skewed right, skewed left, symmetrical, approximately the same.</li> <li>• Understand the shape of the data can be skewed left, skewed right or symmetrical.</li> <li>• Understand how to calculate the mean, median, and range.</li> <li>• Understand if the mean and median are approximately the same, the shape of the data is symmetric.</li> <li>• Understand if the mean is less than the median, the shape of the data is skewed left.</li> <li>• Understand if the mean is greater than the median, the shape of the data is skewed right</li> </ul>
<p>MA.912.DP.2.4</p>	<p>Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data</p> <p><b>Access Point</b>  MA.912.DP.2.AP.4 Fit a linear function to bivariate</p>

numerical data that suggests a linear association and interpret the slope and  $y$ -intercept of the model.

**Essential Understandings:**

- Understand the following terms and vocabulary: linear function, linear association, data models, linear fit, bivariate numerical data,  $y$ -intercept, slope,  $x$ -axis,  $y$ -axis, steepness, positive, negative, linear model, left, right, upward, downward.
- Understand that a linear association means the data models a line.
- Understand that bivariate data is two numerical values paired with each other (Ex. Ordered pair  $(-2,3)$ )
- Understand if the data models a linear fit, then a linear function in the form of  $y = mx + b$  can be created to fit the data.
- Understand that the linear function may not cross every point given.
- Understand in a linear function the  $y$ -intercept is represented by the variable  $b$ .
- Understand in a linear function the  $y$ -intercept is where the function crosses the  $y$ -axis.
- Understand in a linear function, slope is represented by the variable  $m$ .
- Understand in a linear function, slope measures the steepness of the line.
- Understand in a linear model, if the slope is positive the points on the model will go upward from left to right.
- Understand in a linear model, if the slope is negative the points on the model will go downward from left to right.

MA.912.DP.2.6

Compute the correlation coefficient of a linear model using technology. Interpret the strength and direction of the correlation coefficient.

**Access Point**

MA.912.DP.2.AP.6 Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a

real-world context.

**Essential Understandings:**

- Understand the following terms and vocabulary: line of best fit, residuals, value, observed value, predicted value, graph, point, x-axis, y-axis, linear pattern, positive slope, negative slope, correlation, positive, negative, strong linear fit, weak linear fit, moderate linear, no linear fit, correlation coefficient ( $r$ ), data, linear relationship, strength, direction.
- Understand the line of best fit is the equation of the line that represents the majority of the points on the graph.
- Understand that the residuals are created by subtracting the observed value minus the predicted value.
- Understand that the observed value is the actual point on the graph.
- Understand the predicted value is created using the line of best fit.
- Understand that strength is the measure of how strong the data fits a linear pattern (strong, weak, moderate, no fit).
- Understand that direction is positive (positive slope) or negative (negative slope).
- Understand the correlation measures the strength, the data, fits a line (linear pattern).
- Understand that " $r$ " represents the correlation coefficient.
- Understand that the closer " $r$ " is to  $-1$  or  $1$ , the stronger the data fits a linear relationship between  $x$  and  $y$ .
- Understand that the closer " $r$ " is to  $0$  the weaker the data fits a linear relationship between  $x$  and  $y$ .

MA.912.DP.2.8

Fit a quadratic function to bivariate numerical data that suggests a quadratic association and interpret any intercepts or the vertex of the model. Use the model to solve real-world problems in terms of the context of the



	<p>data.</p> <p><b>Access Point</b></p> <p>MA.912.DP.2.AP.8 Given a scatter plot, select a quadratic function that fits the data the best.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: scatter plot, quadratic function, graph, ordered pairs, data.</li> <li>• Understand that a quadratic function uses the following rule: <math>y = ax^2 + bx + c</math>.</li> <li>• Understand to determine which function fits the data that is graphed, choose ordered pairs from the graph to plug into the given functions to determine which function is true.</li> </ul>
MA.912.DP.2.9	<p>Fit an exponential function to bivariate numerical data that suggests an exponential association. Use the model to solve real-world problems in terms of the context of the data.</p> <p><b>Access Point</b></p> <p>MA.912.DP.2.AP.9 Given a scatter plot, select an exponential function that fits the data the best.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: scatter plot, exponential function, graph, ordered pairs, data.</li> <li>• Understand that an exponential function uses the following rule: <math>y = a^x</math>.</li> <li>• Understand to determine which function fits the data that is graphed, choose ordered pairs from the graph to plug into the given functions to determine which function is true.</li> </ul>
<b>MA.912.DP.3 Solve problems involving categorical data.</b>	
MA.912.DP.3.1	<p>Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.</p> <p><b>Access Point</b></p>

	<p>MA.912.DP.3.AP.1 When given a two-way frequency table summarizing bivariate categorical data, identify joint and marginal frequencies.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: two-way frequency table, bivariate categorical data, marginal frequencies, joint frequencies, attributes, characteristics, classify.</li> <li>• Understand that a two-way table has two types of frequencies: joint frequencies (numbers inside the two-way table) and marginal frequencies (totals in the margins or edge of table).</li> <li>• Understand that categorical data is data that is classified by attributes or characteristics (Ex. Favorite color, type of car, number on a sports jersey) .</li> <li>• Understand that bivariate data has two characteristics or attributes. (Ex. Height and weight).</li> </ul>
MA.912.DP.3.2	<p>Given marginal and conditional relative frequencies, construct a two-way relative frequency table summarizing categorical bivariate data.</p> <p><b>Access Point</b>  MA.912.DP.3.AP.2 Given the marginal relative frequencies and a partially completed two-way table, calculate one missing value per row and/or per column.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: marginal relative frequencies, frequency, two-way table, marginal frequency, margins, vertical, horizontal, relative frequency, row, column, relative frequency, joint relative frequency, percentage, addition, subtraction, value.</li> <li>• Understand that a frequency in a two-way table is the number of times a value appears in a set of data.</li> <li>• Understand that a marginal frequency is the values that appear in the margins of a table (vertical and horizontal totals).</li> </ul>

	<ul style="list-style-type: none"> <li>• Understand that a relative frequency is the percentage of the total value from a data set.</li> <li>• Understand that a marginal relative frequency is the percentage of the total that appear in the margins of a table.</li> <li>• Understand to calculate a missing row or column value, add joint relative frequencies (values inside the table) and subtract that value from the marginal relative frequency.</li> </ul>
MA.912.DP.3.3	<p>Given a two-way relative frequency table or segmented bar graph summarizing categorical bivariate data, interpret joint, marginal and conditional relative frequencies in terms of a real-world context.</p> <p><b>Access Point</b>  MA.912.DP.3.AP.3 Given a segmented bar graph summarizing categorical bivariate data, select the interpretation in terms of a real-world context.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: segmented bar graph, categorical bivariate data, stacked bars, attributes, characteristics, horizontal, vertical, category.</li> <li>• Understand that a segmented bar graph is a chart that compares two or more categories with horizontal or vertical stacked bars.</li> <li>• Understand that a stacked bar is a single bar within a bar graph that displays more than one attribute (gender, age group, economic class, etc.).</li> <li>• Understand that categorical data is data that is labeled by attributes or characteristics (colors, places, cars etc.).</li> <li>• Understand that categorical bivariate data is data collected from two categories (color/gender, height/weight, etc.)</li> </ul>
<b>MA.912.DP.4 Use and interpret independence and probability.</b>	
MA.912.DP.4.1	<p>Describe events as subsets of a sample space using characteristics, or categories, of the outcomes, or as unions, intersections or complements of other events.</p> <p><b>Access Point</b>  MA.912.DP.4.AP.1 Given a sample space, select a subset of the sample space or given two sets, select the union, intersection, or complement of two sets.</p>

	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: sample space, subset, set, union, intersection, complement, Venn Diagram, elements, member.</li> <li>• Understand that a set is a group of objects.</li> <li>• Understand that a sample space is all the possible outcomes in a set of items.</li> <li>• Understand that a subset is a part of the sample space.</li> <li>• Understand that a union is the combination of two or more sets together.</li> <li>• Understand that an intersection of two or more sets is the elements they have in common.</li> <li>• Understand the compliment of two or more sets is any element that is not a member of all given sets.</li> <li>• Understand a Venn Diagram can be used to model the sets.</li> </ul>
MA.912.DP.4.2	<p>Determine if events A and B are independent by calculating the product of their probabilities.</p> <p><b>Access Point</b>  MA.912.DP.4.AP.2 Given the probability of events A and B and the product of their probabilities, select whether the events are independent or not independent.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: probability, independent, product, events.</li> <li>• Understand that independent events are events where the probability of one event does not affect the probability of another event. (Flipping a coin and rolling a die are independent because flipping the coin does not affect the probability of rolling a die).</li> <li>• Understand that events are independent if the probability of A multiplied by the probability of B is equal to the probability of A and B (<math>P(A)P(B) = P(A \text{ and } B)</math>).</li> </ul>
MA.912.DP.4.3	<p>Calculate the conditional probability of two events and interpret the result in terms of its context.</p>
	<p><b>Access Point</b>  MA.912.DP.4.AP.3 Given the probability of two events, P(A and B) and P(A), in decimal form, select the conditional probability of the two events <math>\{P(A \text{ and } B) / P(A)\}</math>.</p>

	$P(B A) = \frac{P(A \text{ and } B)}{P(A)}$
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: probability, events decimal form, conditional probability.</li> <li>• Understand that conditional probability is the probability of an event occurring given that we have prior information. (The probability of surviving the sinking of the titanic given that the person is a female. Females were put in the lifeboats first therefore more likely to survive the accident.)</li> <li>• Understand that the conditional probability of B given A is calculated by dividing the probability of A and B by the probability of A. (<math>P(B A) = \frac{P(A \text{ and } B)}{P(A)}</math>).</li> </ul>
MA.912.DP.4.6	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations
	<p><b>Access Point</b>  MA.912.DP.4.AP.6 Recognize the concept of independence in everyday situations.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: independence, events, probability.</li> <li>• Understand that independent events are events where the probability of one event does not affect the probability of another event. (Flipping a coin and rolling a die are independent because flipping the coin does not affect the probability of rolling a die).</li> </ul>
MA.912.DP.4.7	Apply the addition rule for probability, taking into consideration whether the events are mutually exclusive, and interpret the result in terms of the model and its context.
	<p><b>Access Point</b>  MA.912.DP.4.AP.7 Given the probability of two mutually exclusive events in decimal form, use the addition rule for mutually exclusive probabilities: <math>P(A \text{ or } B) = P(A) + P(B)</math>.</p>
	<p><b>Essential Understandings:</b></p>

	<ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: probability, mutually exclusive events, decimal form, addition rule.</li> <li>• Understand that mutually exclusive events are events that cannot occur at the same time. (A person cannot be overweight and underweight at the same time).</li> <li>• Understand that the probability of events A or B is calculated by adding the probability of A and the probability of B (<math>P(A \text{ or } B) = P(A) + P(B)</math>).</li> </ul>
MA.912.DP.4.8	Apply the general multiplication rule for probability, taking into consideration whether the events are independent, and interpret the result in terms of the context.
	<p><b>Access Point</b>  MA.912.DP.4.AP.8 Given the probability of two independent events in decimal form, use the multiplication rule for independent probabilities: <math>P(A \text{ and } B) = P(A)P(B)</math>.</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: independent events, probability, decimal form, multiplication rule.</li> <li>• Understand that events are independent if the probability of A multiplied by the probability of B is equal to the probability of A and B (<math>P(A)P(B) = P(A \text{ and } B)</math>).</li> </ul>
MA.912.DP.5.11	Evaluate reports based on data from diverse media, print and digital resources by interpreting graphs and tables; evaluating data-based arguments; determining whether a valid sampling method was used; or interpreting provided statistics.
	<p><b>Access Point</b>  MA.912.DP.5.AP.11 Given a graph representing data, select whether the graph is misleading or not (i.e., scale on x and y axis not consistent, circle graph does not add up to 100%; missing title or title doesn't represent data; or bar widths on bar graph are inconsistent).</p>
	<p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: graph, data, misleading.</li> <li>• Understand that a graph can be misleading (i.e.,</li> </ul>

	scale on x and y axis not consistent, circle graph does not add up to 100%; missing title or title doesn't represent data; or bar widths on bar graph are inconsistent).
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### 9-12 Trigonometry Strand

<b>MA.912.T.1 Define and use trigonometric ratios, identities or functions to solve problems.</b>	
MA.912.T.1.1	Define trigonometric ratios for acute angles in right triangles.
	<b>Access Point</b> MA.912.T.1.AP.1 Select a trigonometric ratio for acute angles in right triangles limited to sine or cosine.
	<b>Essential Understandings:</b> <ul style="list-style-type: none"> <li>• Understand the following terms and vocabulary: triangle, opposite side, adjacent side, hypotenuse, sine (sin), cosine (cos), right triangle, ratio, trigonometric ratio, length.</li> <li>• Understand that a right triangle is a triangle that has one right angle.</li> <li>• Understand when given an angle on a right triangle, identify the opposite side and the adjacent side.</li> <li>• Understand when given a right triangle, identify the hypotenuse.</li> <li>• Understand the trigonometric ratio of sine in a right triangle is <math>\sin \theta = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}}</math></li> <li>• Understand the trigonometric ratio of cosine in a right triangle is <math>\cos \theta = \frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}</math></li> </ul>
MA.912.T.1.2	Solve mathematical and real-world problems involving right triangles using trigonometric ratios and the Pythagorean Theorem.
	<b>Access Point</b> MA.912.T.1.AP.2 Given a mathematical and/or real-world problem involving right triangles, solve using trigonometric ratio or the Pythagorean Theorem.

### Essential Understandings:

- Understand the following terms and vocabulary: triangle, opposite side, adjacent side, hypotenuse, leg of a triangle, sine (sin), cosine (cos), tangent (tan), cotangent (cot), cosecant (csc), secant (sec), Pythagorean Theorem, right triangle, ratio, trigonometric ratio, length.
- Understand that a right triangle is a triangle that has one right angle.
- Understand when given an angle on a right triangle, identify the opposite side and the adjacent side.
- Understand when given a right triangle, identify the hypotenuse.
- Understand the trigonometric ratio of sine in a right triangle is  $\sin \theta = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$
- Understand the trigonometric ratio of cosine in a right triangle is  $\cos \theta = \frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$
- Understand the trigonometric ratio of tangent in a right triangle is  $\tan \theta = \frac{\text{length of the opposite side}}{\text{length of the adjacent side}}$
- Understand the trigonometric ratio of cotangent in a right triangle is  $\cot \theta = \frac{\text{length of the adjacent side}}{\text{length of the opposite side}}$
- Understand the trigonometric ratio of cosecant in a right triangle is  $\csc \theta = \frac{\text{length of the hypotenuse}}{\text{length of the opposite side}}$
- Understand the trigonometric ratio of secant in a right triangle is  $\sec \theta = \frac{\text{length of the hypotenuse}}{\text{length of the adjacent side}}$
- Understand the Pythagorean Theorem is the following:  $a^2 + b^2 = c^2$  where  $a$  and  $b$  are lengths of the legs of the right triangle and  $c$  is the length of the hypotenuse

### 9-12 Logic and Theory Strand



**MA.912.LT.4 Develop an understanding of the fundamentals of propositional logic, arguments and methods of proof.**

MA.912.LT.4.3	<p>Identify and accurately interpret “if...then,” “if and only if,” “all” and “not” statements. Find the converse, inverse and contrapositive of a statement.</p> <p><b>Access Point</b></p> <p>MA.912.LT.4.AP.3 Identify and accurately interpret “if...then,” “if and only if,” “all” or “not” statements.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"><li>• Understand the following terms and vocabulary: relationships, logical, interpret, if...then, if and only if, all, not</li><li>• Understand that “if...then”, “if and only if”, “all” or “not” have logical relationships.</li></ul> <p>Ex. If a figure is a square, then it is a quadrilateral. <math>a + 4 = 6</math>, if and only if <math>a = 2</math>. All triangles have three sides Not all rectangles are squares</p>
MA.912.LT.4.10	<p>Judge the validity of arguments and give counterexamples to disprove statements.</p> <p><b>Access Point</b></p> <p>MA.912.LT.4.AP.10 Select the validity of an argument or give counterexamples to disprove statements.</p> <p><b>Essential Understandings:</b></p> <ul style="list-style-type: none"><li>• Understand the following terms and vocabulary: premise, argument, conclusion, valid, invalid, counterexample, true, false.</li><li>• Understand that an argument is made up of a premise (initial statement) followed by a conclusion.</li><li>• Understand that an argument is valid if and only if the premise is true and the conclusion is true.</li></ul> <p>Ex. Valid argument</p> <ol style="list-style-type: none"><li>1. If Bob has a turtle, then Bob has a reptile.</li><li>2. Bob has a turtle.</li><li>3. Therefore, Bob has a reptile.</li></ol>

Invalid argument (invalid argument because the premise and conclusion are false)

1. All rectangles are squares.
2. All squares are triangles.
3. Therefore, all rectangles are triangles.

Example

All cats are hairy.

Counterexample: This is invalid because there are hairless cats.